STA 301.1 Reg. No. ....

CREDIT BASED THIRD SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2012

STATISTICS

DISTRIBUTIONS AND ESTIMATION THEORY

Time: 3 Hrs **Max. Marks: 80**

PART - A

Answer any TEN of the following: **2X10=20**

1. a) If 'X' has uniform distribution over the range $(0, \theta)$, find the M.G.F.

   b) Define Cauchy distribution with parameters λ and θ.

   c) Obtain the mean of X with p.d.f. $f(x) = \theta e^{-x\theta}$ $x > 0$ $\theta > 0$

   d) Write down the density function of Beta distribution of second kind

   e) If Z is a S.N.V., write down the density function of Z^2

   f) If $F(x)$ is the distribution function of X find the distribution of $Y = F(x)$

   g) Find the distribution function of n^{th} order statistics.

   h) Write down the p.d.f. of F variate with (n_1, n_2) d.f

   i) If T is unbiased for θ, show tha$\sqrt{T}$ is biased for $\sqrt{\theta}$.

   j) Define consistency of estimator.

   k) If $x_1 x_2 \ldots x_n$ is a random sample $U(0, \theta)$ find the moment estimator of θ.

   l) What is meant by interval estimation?

PART – B

Answer any TWO of the following: **10x2=20**

2. a) State and prove lack of memory property of exponential distribution.

   b) Derive the MGF of Gamma distribution with parameters a & n. **(5+5)**

3. a) Show that linear combination of independent Normal variates is a Normal variate.

   b) Derive the expression for even order central moments of Normal distribution. **(5+5)**

4. a) Define Beta variate of first kind. Derive its mean and variance.

   b) Find median of Cauchy distribution with parameter λ and θ. **(5+5)**

Answer any TWO of the following: **10x2=20**

5. a) If x and y are independent exponential variates with respective means θ_1 and θ_2 find the distribution of $x + y$.

   b) If x and y are independent gamma variates with respective parameters m and n. Find the distribution of $\frac{x}{x+y}$ **(5+5)**

6. a) Derive the p.d.f. of Chi-square variates with 'n' degrees of freedom.

   b) Obtain mean and variance of t variate. **(5+5)**

7. a) Derive the mean of F distribution.

   b) Show that if sampling from normal population with variance σ^2, the variate ns^2/σ^2 has χ^2 distribution with $(n - 1)$ d.f. provided $\bar{x}$ and s^2 are independent, where $\bar{x}$ and s^2 are the sample mean and variance respectively. **(5+5)**

Answer any TWO of the following: **10x2=20**

8. a) Show that when sampling from normal distribution sample variance is a biased estimator of population variance but asymptotically unbiased.

   b) Let x_1, x_2 be independent random sample from $P(\lambda)$. Show that $x_1 + x_2$ is sufficient for λ **(5+5)**

9. a) Find the m.l.e. of θ in the beta distribution of the first kind with parameters θ and 1.

   b) $x_1 x_2 ... x_n$ are i.i.d. $B(n, p)$ where both n and p and unknown. Find the moment estimators of n and p. **(5+5)**

10. a) Derive the $100(1 - \alpha)$ % central confidence interval for the ratio of variances of two independent normal population with known means.

    b) Derive the $100(1 - \alpha)$ % central confidence interval for the difference in proportions of two independent populations based on two samples of large sizes. **(6+4)**

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STA 301.1 Reg. No. ....

CREDIT BASED THIRD SEMESTER B.Sc. DEGREE EXAMINATION
OCTOBER 2013
STATISTICS
DISTRIBUTIONS AND ESTIMATION THEORY

Time: 3 Hrs **Max. Marks: 80**

PART - A

Answer any TEN of the following: **2X10=20**

1. a) If 'X' has uniform distribution over the range (0, 1), find the mean and variance.

   b) If X is a continuous random variable with *c.d.f.* $(F(x)$, then show that $Y = log_e F(x)$ is an exponential random variable.

   c) Obtain the mean of X with p.d.f. $f(x) = \frac{1}{\theta} e^{-x/\theta} \quad x > 0 \quad \theta > 0$

   d) Write down the p.d.f. of Beta variate of the first kind.

   e) If X is a Normal variate with mean μ and S.D. σ, then write down the p.d.f. of $\left(\frac{x-\mu}{\sigma}\right)^2$

   f) Find the distribution function of first order statistics.

   g) Show that for normal distribution all odd moments about mean vanish.

   h) Write down the p.d.f. of students 't' variate with 'n' degrees of freedom.

   i) If t is unbiased for θ, show that $\sqrt{t}$ is biased for $\sqrt{\theta}$.

   j) Distinguish between an estimator and an estimate.

   k) Explain the concept of relative efficiency.

   l) What is meant by interval estimation?

PART – B

Answer any TWO of the following: **10x2=20**

2. a) Obtain moment generating function and cumulant generating function of the distribution with p.d.f. f(x): $\theta e^{-x\theta}$ $0 < x < \infty$, $\theta > 0$ and hence obtain β_1 and β_2.

   b) Derive the MGF of Gamma distribution with parameters α and n. **(5+5)**

3. a) Find the MGF and Cumulant generating function of Normal distribution. Find β_1 and β_2 using Cumulant generating function.

   b) State and prove additive property of Normal variates. **(5+5)**

4. a) Define Beta distribution of the second kind. Find its mean and variances.

b) Find median of Cauchy distribution with parameters λ and θ. **(5+5)**

Answer any TWO of the following: **10x2=20**

5. a) If X_1 and X_2 are two independent gamma variates with parameters λ_1 and λ_2 respectively. Find the distribution of X_1/X_2. Identify the distribution.

b) If X_1 and X_2 are two independent standard normal variates find the distribution of $\left(\frac{X_2-X_1}{2}\right)^2$ **(5+5)**

6. a) Derive the p.d.f. of Chi-square variates with 'n' degrees of freedom.

b) Derive the even order central moments of 't' distribution with 'n' degrees of freedom. **(5+5)**

7. a) Derive the mode of F distribution.

b) If F is Snedecor's F-variate with (n_1, n_2) degrees of freedom obtain the distribution of 1/F. **(5+5)**

Answer any TWO of the following: **10x2=20**

8. a) If $x_1\, x_2 \dots x_n$ is a random sample from $U(-\theta, \theta)$ distribution show that $T_n = \max\,(x_1, x_2 \dots x_n)$ is biased for θ. Is Tn asymptotically unbiased?.

b) If $x_1\, x_2 \dots x_n$ is a random sample from $U(-\theta, \theta)$ distribution show that $T_n = \sum x_i$ is sufficient statistic for θ. **(5+5)**

9. a) If $x_1\, x_2 \dots x_n$ is a random sample from the exponential distribution with density $f(x) = \frac{1}{\theta} e^{-x/\theta} \quad x > 0 \quad \theta > 0$, find the m.l.e. of θ

b) If $x_1\, x_2 \dots x_n$ are i.i.d. $B(n, \theta)$ where both n and p and unknown. Find the moment estimators of n and p. **(5+5)**

10. a) Derive the 100(1 - α) % central confidence interval for difference in means of two independent normal populations with unknown but common variance.

b) Obtain the confidence interval for the population proportion based on the large sample. **(6+4)**

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STA 301.1 Reg. No. ....

CREDIT BASED THIRD SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2014

STATISTICS

DISTRIBUTIONS AND ESTIMATION THEORY

Time: 3 Hrs **Max. Marks: 80**

PART - A

Answer any TEN of the following: **2x10=20**

1. a) Obtain the distribution function of a rectangular distribution over the range (a, b).

   b) Give two real life situations wherein the exponential distribution is appropriate.

   c) If $x_1, x_2 \ldots\ldots x_n$ is a random sample from normal with mean μ and variance σ^2. What is the probability distribution of $\bar{x}$?

   d) State the relation between Normal, Chi square, t and F distribution.

   e) Distinguish between parameter and statistic.

   f) Show that $2\bar{x}$ is an unbiased estimate of θ for $f(x,\theta) = 1/\theta$. $0 \le x \le \theta$.

   g) Give an example of an estimator which is consistent but biased.

   h) Mention any four properties of MLE.

   i) Explain the concept of efficiency.

   j) What is meant by interval estimation?

   k) When Beta distribution of first kind is symmetric?

   l) Find the distribution of first order statistic.

PART – B

Answer any TWO of the following: **10x2=20**

2. a) If X follows $U\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ find the distribution of Y = tan X and identify the distribution.

   b) If X follows exponential distribution with density function, $f(x) = \frac{1}{\beta} e^{-\frac{(x-\alpha)}{\beta}}$ $x \ge \alpha$ .Find the variance. **(5+5)**

3. a) State and prove memory less property of exponential distribution.

   b) For a normal distribution with parameters μ and σ^2, find mean deviation from mean. **(5+5)**

4. a) Find the median of Cauchy distribution with parameter λ and θ.

b) If $x \sim \beta_1(m,n)$, derive the distribution of $Y = \frac{x}{1-x}$. **(4+4)**

Answer any TWO of the following: **10x2=20**

5. a) If x follows $N(\mu,\sigma^2)$, find the distribution of $\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2$.

b) If $x \sim \beta_1(m,n), y \sim \gamma(\alpha, m+n)$. Find the distribution of $z = xy$ and identify the distribution. **(5+5)**

6. a) Derive the p.d.f. of Chi square variate with n degrees of freedom.

b) Obtain variance of t variate. **(5+5)**

7. a) Derive the mean of F distribution.

b) Show that sampling form Normal population with variance σ^2, the variate ns^2/σ^2 has Chi square distribution with (n – 1) degrees of freedom provided $\bar{x}$ and s^2 are independent where $\bar{x}$ and s^2 are the sample mean and variance respectively. **(5+5)**

Answer any TWO of the following: **10x2=20**

8. a) Show that when sampling from Normal distribution, sample variance is a biased estimator of population variance but asymptocally unbiased.

b) Let X_1, X_2 be iid $P(\theta)$ random variables, show that $X_1 + 2X_2$ is not sufficient estimate of θ **(5+5)**

9. a) Find the MLE of θ in Bernoulli distribution based on a random sample of size n.

b) Describe moment method of estimation. **(5+5)**

10. a) Obtain the large sample confidence interval for the difference between the proportions of two populations.

b) Derive $100\,(1-\alpha)\,\%$ central confidence internal for the difference in means of two independent normal population with common unknown variance. **(5+5)**

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STA 301.1 Reg. No. ....

CREDIT BASED THIRD SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2015

STATISTICS

DISTRIBUTIONS AND ESTIMATION THEORY

Time: 3 Hrs **Max. Marks: 80**

PART - A

Answer any TEN of the following: **2x10=20**

1. a) Find mean and variance of uniform distribution over the vary $(-\theta, \theta)$

   b) If X has th p.d.f. $f(x) = \frac{1}{\theta} e^{-x/\theta}$ $x \geq 0$ $\theta > 0$. Find its distribution function.

   c) Show that uniform distribution is the particular case of the beta distribution of first kind.

   d) State the relation between normal, Chi square, t and F distribution.

   e) Write down the p.d.f. of beta distribution of first kind.

   f) If $x_i \sim N(G_1, G_i^2,$ $i = 1....n$ independent what is the mean of $\sum \left(\frac{x_i - G_i}{\sigma_i} \right)$?

   g) If $X \sim t_{(1)}$ then what is the p.d.f. of X?

   h) The mode of x^2 distribution is 6. What are its mean and variance?

   i) If T is unbiased for θ. Show that T^2 is biased for θ^2.

   j) Define sufficiency of an estimation.

   k) Define relative efficiency of an estimator.

   l) What is meant by interval estimation?

PART – B

Answer any TWO of the following: **10x2=20**

2. a) Find the median of Cauchy distribution with parameter λ and θ.

   b) Derive mean and variance of gamma distribution with parameters d, n. **(5+5)**

3. a) Find the mode of normal distribution.

   b) Derive the expression of even order central moments of normal distribution. **(5+5)**

4. a) Find mean and variance of beta distribution of second Kxd.

   b) If $f(x) = \theta e^{-\theta x}$ $x \geq 0$ $\theta > 0$ then show that for every $a \geq 0$ $P[y \leq x / x \geq a] = p(x \leq x)$.

   (4+4)

Answer any TWO of the following: **10x2=20**

5. a) If X and Y are independent exponential variates with respective parameter θ, find the distribution of X + Y.

   b) If X and Y are independent gamma variates with respective parameters m and n, find the distribution of $x \mid x+y$. (5+5)

6. a) Find mode of X^2 variate with 'n' d.f..

   b) Derive the density fundtion of 't' variate with 'n' d.f. (5+5)

7. a) Derive variance of 'F' distribution assuming mean.

   b) Find mean and variance of t distribute with n d.f. (5+5)

Answer any TWO of the following: **10x2=20**

8. a) Prove that sample mean $\overline{x}$ is consistent for θ in $N(\theta, \sigma^3)$.

   b) If $x_1, x_2, ...x_n$ in a random sample from Bernoulli $B(\theta)$ distribution. Show that $T_n = \Sigma x_i$ in a sufficient statistic force. (5+5)

9. a) If $x_1, x_2, ...x_n$ be i.j.d. r.o.s. with common p.d.f.'s $f(x, 0) = \frac{\theta^2}{\lceil \alpha} e^{-\theta x} x^{2-1} x > 0$ and α is known. Find the m.l.e. of θ.

   b) Find the moment estimators of the parameters of the uniform distribution over the interval $((\theta_1, \theta_2)$ (5+5)

10. a) Derive the $100\,(1-\alpha)\,\%$ central confidence internal for the difference in means of two independent normal population with common unknown variance.

    b) Obtain the confidence interval for the population proportions based on a large sample. (5+5)

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STA 301.1 Reg. No. ....

CREDIT BASED THIRD SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2016

STATISTICS

DISTRIBUTIONS AND ESTIMATION THEORY

Time: 3 Hrs **Max. Marks: 80**

PART - A

Answer any TEN of the following: **2x10=20**

1. a) A random variable x is distributed uniformly over the interval (a, b) State its mean and variance.

   b) Define Cauchy distribution with two parameters.

   c) State its relation of Beta variate with Gamma variate.

   d) If x has p.d.f. $f(x) = e^{-x}, \quad x \geq 0$ Find its mean.

   e) Write down any two important characteristics of Normal distribution.

   f) Find the distribution function of n^{th} order statistic.

   g) If X and Y are independent normal variates, what is the distribution of X+ Y?

   h) Write down the p.d.f. of Snedecor's F distribution with (m, n) degrees of freedom.

   i) Define unbiasedness and asymptotic unbiasedness of an estimator.

   j) Mention any two properties of maximum likelyhood estimators.

   k) Find the moment estimator of the parameter θ of the uniform distributon over the interval $(-\theta, \theta)$.

   l) What are Confidence intervals?

PART – B

Answer any TWO of the following: **10x2=20**

2. a) Obtain mean and variance of an exponential distribution whose p.d.f. is

   $$f(x, \theta, \alpha) = \frac{1}{\theta} e^{-(x-\alpha)/\theta} \quad x \geq \alpha \quad \theta > 0$$

   b) Find the median of Cauchy distribution with parameters λ and θ. **(5+5)**

3. a) Derive the expression for even order central moments of a Normal distribution.

   b) Find the median of Normal distribution. **(6+4)**

4. a) Define Beta distribution of first kind, find its mean and variance.

   b) Find the MGF of Gamma variate with parameter 'n'. **(5+5)**

Answer any TWO of the following: **10x2=20**

5. a) If X and Y are two independent gamma variates with parameters m and n respectively find the distribution of $X/(X+Y)$ identify the distribution.

   b) If X_1, X_2 are random observations from an exponential distribution with p.d.f. $f(x,\theta)=\theta e^{-x\theta}$ $x>0$ $\theta>0$. Find the distribution of X_1+X_2. (6+4)

6. a) Derive students 't' distribution with 'n' degrees of freedom.

   b) Obtain mean and variance of Chi-square distribution with n degrees of freedom. (6+4)

7. a) Derive the expression for even order central moments of 't' distribution.

   b) Deduce the mean of F distribution. (6+4)

Answer any TWO of the following: **10x2=20**

8. a) Show that when sampling from normal population sample standard deviation is asymptotically unbiased and consistent for population standard deviation.

   b) Show that the sum of items of a random sample of size n from an exponential distribution with p.d.f. $f(x,\theta)=\frac{1}{\theta}e^{-x/\theta}$ $x>0$ is sufficient for θ. (6+4)

9. a) Determine the m.l.e. of the parameter of Poisson distribution based on a sample of size n.

   b) $x_1, x_2, \ldots x_n$ is a random sample from $N(0,\theta)$. Find the moment estimator of θ. Is it uniquely determined? (5+5)

10. a) Obtain $100(1-\alpha)\%$ central confidence interval for the ratio of variance of two independent normal populations with known means.

    b) Derive the $100(1-\alpha)\%$ central confidence interval for difference in proportions of two independent populations based on 2 samples and large sizes. (6+4)

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