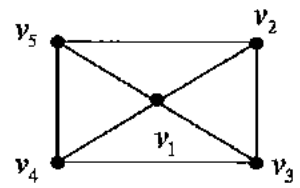
UNIT-IV

8. a) Prove that every tree with two or more vertices is 2 chromatic. **(6)**

b) Prove that a graph with n vertices is a tree if and only if its chromatic polynomial is $P_n(\lambda)=\lambda(\lambda-1)^{n-1}$. **(6)**

c) Write the chromatic polynomial of the following graph. **(6)**



9. a) Prove that a graph on n vertices is a complete graph if and only if its chromatic polynomial is $P_n(\lambda)=\lambda(\lambda-1)(\lambda-2)\cdots(\lambda-n+1)$. **(6)**

b) Prove that a graph with at least one edge is two chromatic if and only if it has no circuit of odd length. **(6)**

c) Let a and b be two non adjacent vertices in a graph G. Let G' be a graph obtained by adding an edge between a and b. Let G'' be the simple graph obtained from G by fusing the vertices a and b together and replacing sets of parallel edges with a single edge. Prove that $P_n(\lambda)$ of $G = P_n(\lambda)$ of $G' + P_{n-1}(\lambda)$ of G''. **(6)**

UNIT-V

10. a) Prove that a digraph G is an Euler digraph if and only if it is connected and balanced. **(6)**

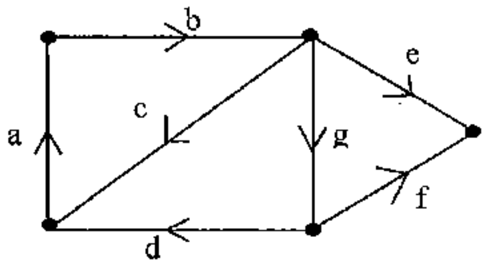
b) Let A_f be the reduced incidence matrix of a connected digraph G. Then prove that the number of spanning trees in the graph equals the value of determinant $(A_f \cdot A_f^T)$. **(6)**

c) Define incidence matrix for digraphs. Draw the digraph for the given incidence matrix. **(6)**

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{pmatrix} 1 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix} \end{array}$$

11. a) Prove that arborescence is a tree in which every vertex other than the root has an in-degree of exactly one. **(6)**

b) Write a circuit matrix of the digraph **(6)**



c) Show that determinant of every square submatrix of the incidence matrix of a digraph is 1, -1, or 0. **(6)**

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MAT 602.3 Reg. No. ....

CREDIT BASED SIXTH SEMESTER B.Sc. DEGREE EXAMINATION APRIL 2016

MATHEMATICS

PAPER VIII: GRAPH THEORY

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.

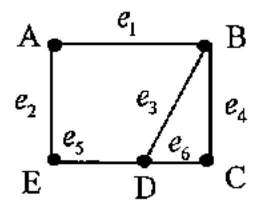
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A **3x10=30**

1. a) Prove that the maximum number of edges in a simple graph is $\frac{n(n-1)}{2}$

b) In a binary tree of n vertices prove that the number of pendent vertices is $p=\frac{n+1}{2}$

c) List any 3 paths from a vertex 'A' to vertex 'C' in the following graph



d) Prove that edge connectivity of a graph G cannot exceed the degree of the vertex with the smallest degree in G.

e) Define the terms

i) Fundamental cutset

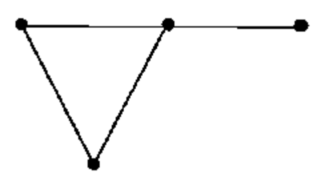
ii) Planar graph

f) If G is a simple connected planar graph, with n vertices, e edges and f regions, then prove that $e \ge \frac{3}{2} f$

g) Draw the graph whose adjacency matrix is

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

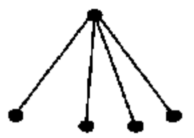
h) Draw the dual of the graph

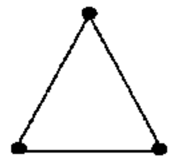


i) Define path matrix for the vertex pair (x, y) of a graph G.

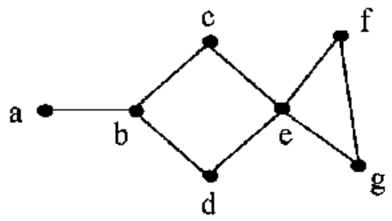
j) State how to identify degree of a vertex and isolated vertex in an incidence matrix of a graph?

k) Find the chromatic number of the following graphs.

i) 

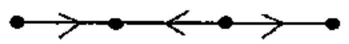
ii) 

l) List any 3 maximal independent sets in the following graph G.



m) Define arborescence.

n) Write the adjacency matrix of



o) Draw an example for an Euler digraph

PART - B

UNIT-I

2. a) Prove that a connected graph G is an Euler graph if and only if all vertices of G are of even degree. **(6)**

b) Prove that a tree with n vertices has $n-1$ edges. **(6)**

c) If a graph has exactly 2 vertices of odd degree then prove that there exists a path joining these two vertices. **(6)**

3. a) Prove that a simple graph with n vertices and k components have at most $\frac{(n-k)(n-k+1)}{2}$ edges. **(6)**

b) Prove that every tree has at least two pendent vertices. **(6)**

c) Show that the graph is a tree if and only if it is minimally connected. **(6)**

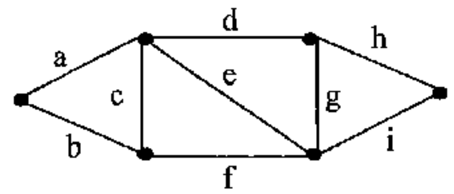
UNIT-II

4. a) Prove that Kuratowski's first graph K_5 is nonplanar. **(6)**

b) Prove that in a connected planar graph with n vertices and e edges, there are $e-n+2$ regions. **(6)**

c) Prove that in a connected graph G, any minimal set of edges containing at least one branch of every spanning tree of G is a cutset. **(6)**

5. a) Define fundamental cutset in a connected graph and list all fundamental cutsets for the following graph with respect to the spanning tree $T=\{a,c,f,g,h\}$ **(6)**



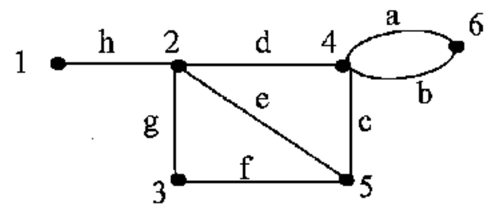
b) Prove that graph K_{33} has no dual. **(6)**

c) Prove that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane. **(6)**

UNIT-III

6. a) Prove that ring sum of two circuits in a graph G is either a circuit or an edge disjoint union of circuits. **(6)**

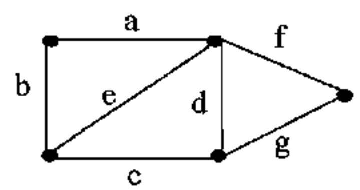
b) Write the path matrix P(1,4) of the following graph **(6)**



c) If A(G) is the incident matrix of a connected graph G with n vertices, show that rank of A(G) $= n-1$ **(6)**

7. a) Show that in a vector space of a graph the circuit subspace W_Γ and the cutset subspace W_S are orthogonal to each other. **(6)**

b) Write the cutset matrix for the following graph. **(6)**



(6)

c) Let A and B respectively, be the incidence and the circuit matrix of a self loop free graph whose columns are arranged using the same order of edges. Prove that $A\cdot B^T = B\cdot A^T \equiv 0 \pmod 2$. **(6)**