MAT 501 Reg. No. ....

CREDIT BASED FIFTH SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2012

MATHEMATICS

PAPER V: ALGEBRA AND DIFFERENTIAL EQUATIONS

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A

3x10=30

1. a) Solve $(D^2+4)y = cos2x$

b) Solve $(D^2-2D+1)y = 2e^x$

c) Find the complementary function of $(D^2-8D+9)y = sin5x$

d) Transform $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = x^2$ into a differential equation with constant coefficients by the substitution $z = log\,x$.

e) Reduce the equation $y_2 - 4xy_1 + (4x^2-3)y = e^{x^2}$ to normal form.

f) Verify that $y = e^x$ is a solution of the differential equation

$$x\frac{d^2y}{dx^2} + 2(4x-1)\frac{dy}{dx} - (9x-2)y = 0$$

g) Find $L\{t^n\}$ for any positive integer n.

h) Obtain $L\{t \sin kt\}$

i) Prove that $\Gamma(n+1) = n!$ for any positive integer n.

j) Prove that every field is an integral domain.

k) If $\phi: R \to R'$ is a homomorphism with kernel $Ker\phi = (0)$, then prove that ϕ is an Isomorphism.

l) If U is an ideal of a ring R and $1 \in U$ then prove that $U = \mathrm{R}$

m) Define a Euclidean ring.

n) If R is a Euclidean ring and $a \neq 0$ is a unit in R then prove that d(a) = d(1)

o) Find all units in J[i].

PART - B

UNIT-I

2. a) Solve $(D^2-4D-5)y = e^{2x} + 3cos4x$ (6)

b) Solve $(D^2+D+1)y = Sin\,2x$ (6)

c) Solve $(D^3-D^2-D+1)y = 1+x^2$ (6)

3. a) Solve $(3D^2 + D - 14)y = 13e^{2x}$ **(6)**

b) Solve $(D^2 - 4D + 3)y = \sin 3x \cos 2x$ **(6)**

c) Solve $(D^2 + 5D + 6)y = \sin 4x + e^{-2x}$ **(6)**

UNIT-II

4. a) Solve $(D^2 - 2D + 4)y = e^x \sin x$ **(6)**

b) Solve $x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = x + \log x$ **(6)**

c) Solve $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$ by the method of changing the independent variable. **(6)**

5. a) Solve $(D^2 + 2D + 5)y = xe^x$ **(6)**

b) Solve $(x + a)^2 \frac{d^2y}{dx^2} - 4(x + a)\frac{dy}{dx} + 6y = x$ **(6)**

c) Solve $(D^2 + 1)y = \sec x$ by the method of variation of parameters. **(6)**

UNIT-III

6. a) Evaluate (i) $L\{\cos^2 kt\}$ (ii) $L^{-1}\left\{\frac{s}{s^2+8s+16}\right\}$ **(6)**

b) If F (t) has a Laplace transform and if $F(t + w) = F(t)$, then prove that **(6)**

$$L\{F(t)\} = \frac{1}{1 - e^{-sw}} \int_0^w e^{-s\beta} F(\beta) d\beta$$

c) Solve $x''(t) + 4x(t) = 2t - 8$, $x(0) = 1$, $x'(0) = 0$ by the Laplace tansform method. **(6)**

7. a) Prove that $L\{\sin kt - kt \cos kt\} = \frac{2k^3}{(s^2+k^2)^2}$ **(6)**

b) Find the transform of the function Q (t, c) defined by

$$Q(t, c) = \begin{cases} 1 & 0 < t < c \\ -1 & c < t < 2c \end{cases}$$

and Q (t + 2c, c) = Q (t, c) for all t. **(6)**

c) Express in terms of the α function and find $L\{F(t)\}$

if $F(t) = \begin{cases} t^2 & 0 < t < 1 \\ 4 & t > 1 \end{cases}$ **(6)**

UNIT-IV

8. a) If $U,\ V$ are ideals of R and $U + V = \{u + v | u \in U,\ v \in V\}$, then prove that $U + V$ is an ideal in R. (6)

   b) Let $\phi : J\left[\sqrt{2}\right] \to J\left[\sqrt{2}\right]$ be defined by $\phi\left(a + b\sqrt{2}\right) = a - b\sqrt{2}$. Prove that ϕ is an onto homomorphism. Find its kernel. (6)

   c) Let R be a commutative ring with unit element and M be an ideal of R. Prove that M is maximal if and only if $R/_{M}$ is a field. (6)

9. a) Prove that a finite integral domain is a field. (6)

   b) If R is a commutative ring with unit element whose only ideals are (0) and R itself, then prove that R is a field. (6)

   c) If R is the ring of integers and p is a prime number then prove that $U = (p)$ is a maximal ideal of R. (6)

UNIT-V

10. a) Prove that a Euclidean ring possesses unit element. (6)

    b) Let R be a Euclidean ring. Prove that any two elements a and b in R have a greatest common divisor d which can be expressed as $d = \lambda a + \mu b$ for some λ, μ in R. (6)

    c) If f(x) and g(x) are any two nonzero elements of F[x], then prove that $deg\, f(x)\, g(x) = deg\, f(x) + deg\, g(x)$ (6)

11. a) Let R be a Euclidean ring and let A be an ideal in R. Prove that there exists an element $a_0 \in A$ such that $A = \{a_0 x | x \in R\}$ (6)

    b) In a commutative ring R with unit element define the relation 'is an associate of' and show that it is an equivalence relation. (6)

    c) If p is a prime number of the form 4n + 1 then prove that there exist integers a and b such that $p = a^2 + b^2$. (6)

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MAT 502.1 **Reg. No. ……………………………………**

CREDIT BASED FIFTH SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2012

MATHEMATICS

PAPER VI: DISCRETE ATHEMATICS

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A

3x10=30

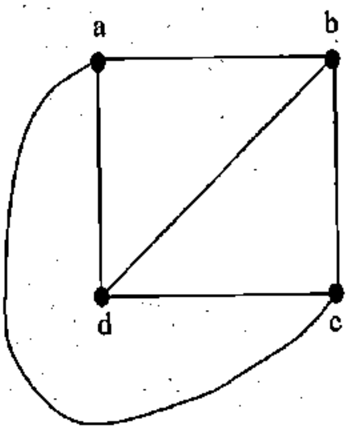
1. a) Define partially ordered set and give an example.

b) If repetitions are allowed, find the number of ways to choose three out of seven days.

c) Prove that $2^n > n^3$ for $n \geq 10$.

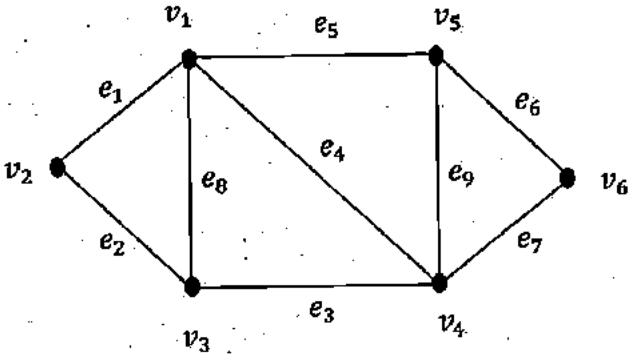
d) Define (i) elementary circuit and
(ii) simple circuit, with examples.

e) When do you say that two undirected graphs are isomorphic? Draw a graph which is isomorphic to the following graph:



f) Prove that the number of odd degree vertices in a graph is always even.

g) Draw any 3 spanning trees of the following graph:



h) Obtain a binary tree for the prefix code $\{1, 01, 000, 001\}$

i) Prove that there is a unique path between every pair of vertices in a tree.

j) Define 'algorithm'. What do you mean by time complexity of the algorithm?

k) Draw the state diagram for the modulo 3 sum of the digits $\{0, 1, 2\}$ in the input sequence.

l) Define tractable and intractable problems.

m) Write the generating function corresponding to the numeric function

$$a_r = 3^{r+2}, r \geq 0.$$

n) Find a particular solution of the difference equation $a_r - 5a_{r-1} + 6a_{r-2} = 1$

o) Define 'forward difference' and 'backward difference' of a numeric function 'a'.

PART - B

UNIT-I

2\. a) In how many ways can a group of 8 people be divided into committees, subject to the constraint that each person must belong to exactly one committee and each committee must contain at least two people. (6)

b) Prove that the set of real numbers between 0 and 1 is not countably infinite. (6)

c) Show that any integer composed of 3^n identical digits is divisible by 3^n. (6)

3\. a) Let $< P, \leq >$ be a partially ordered set. Suppose that the length of the longest chains in P is n, then show that the elements in P can be partitioned into n disjoint antichains. (6)

b) A company purchased 1,00,000 transistors, 50,000 from supplier A, 30,000 from supplier B and 20,000 from supplier C. It is known that 2% of supplier A's transistors are defective, 3% of supplier B's transistors are defective and 5% of the supplier C's transistors are defective.

(i) If a transistor from 100,000 transistors is selected at random, what is the probability that it is not defective?

(ii) Given that a transistor selected from 100,000 transistors at random is defective, what is the probability that it is not from supplier A? (6)

c) Define 'phrase structure grammar'. Provide a step-by-step derivation to obtain the string +001 using the following set of productions:

signed – integer → sign integer
sign → +
sign → –
integer → digit integer
integer → digit
digit → 0
digit → 1 (6)

UNIT-II

4. a) Prove that an undirected graph possesses an Eulerian path if and only if it is connected and has either zero or two vertices of odd degree. (9)

   b) Define 'Hamiltonian path' and prove that there is always a Hamiltonian path in a directed complete graph. (9)

5. a) Let G be a linear graph with n vertices. If the sum of degrees of each pair of vertices in G is n-1 or larger, then prove that there exists an Hamiltonian path in G. (9)

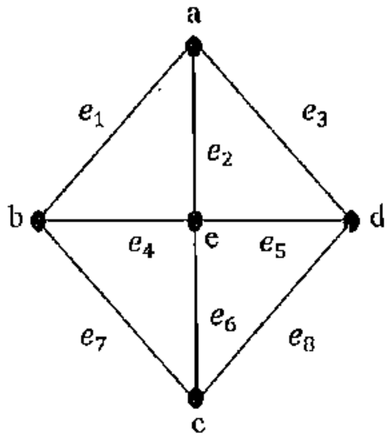
   b) For any connected planar graph, prove with usual notations that $v - e + r = 2$ (9)

UNIT-III

6. a) Prove that every circuit has an even number of edges in common with every cut set. (6)

   b) Prove that a graph with $e = v - 1$ edges that has no circuits is a tree. (6)

   c) Define 'fundamental circuit' in a connected graph G and list all the fundamental circuits with respect to a chosen spanning tree of the graph given below. (6)



7. a) Prove that the number of vertices is one more than the number of edges in a tree. (6)

   b) For a given spanning tree, let $D = \{e_1, e_2, \ldots.,.e_k\}$ be a fundamental cutset in which e_1 is a branch and $e_2, e_3 \ldots.,.e_k$ are chords of the spanning tree. Then prove that e_1 is contained in the fundamental circuits corresponding to e_i, i = 2, 3, ..., k. Also prove that e_1 is not contained in any other fundamental circuits. (6)

   c) Construct a prefix code for the following alphabet, given the respective occurrences:

Letter	a	b	c	d	e	f	g
Occurences	10	5	8	3	6	5	2

(6)

UNIT-IV

8. a) Show that the language $L = \{a^k b^k | k \geq 1\}$ is not a finite state language. **09**

   b) State the algorithm LARGEST2 for finding the largest of n numbers. Also justify it with a formal proof. **09**

9. a) State the algorithm 'BUBBLESORT' for sorting the n numbers $x_2, x_2, \ldots, x_n$. Justify the algorithm with a formal proof. Find its time complexity. **(10)**

   b) Show that the language $L = \{a^k | k = i^2,\ i \geq 1\}$ is not a finite state language. **(8)**

UNIT-V

10. a) Find the homogeneous solution of the difference equation:
$$4a_r - 20a_{r-1} + 17a_{r-2} - 4a_{r-3} = 0$$ **(6)**

    b) Determine the numeric function a_r corresponding to the generating function
$$A(z) = \frac{2}{1-4Z^2}$$ **(6)**

    c) Find the particular solution of the difference equation:
$$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$$ **(6)**

11. a) Write the recurrence relation for the Fibonacci sequence of numbers and find its homogeneous solution. **(6)**

    b) Find the particular solution of the difference equation:
$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)2^r$$ **(6)**

    c) Determine the numeric function corresponding to the generating function
$$A(z) = \frac{2+3z-6z^2}{1-2z}$$ **(6)**

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MAT 502.4 Reg. No. ....

CREDIT BASED FIFTH SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2012

MATHEMATICS

PAPER VI: LINEAR PROGRAMMING

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A

3x10=30

1. a) Define a line segment.

b) Define a Convex set in R^n

c) Pivot $a_{22} = 3$ in the following table.

$$\begin{array}{cc|c|l} x_1 & x_2 & -1 & \\ \hline 2 & 3 & 6 & = -t_1 \\ 1 & 3 & 2 & = -t_2 \\ \hline 0 & 2 & 1 & = f \\ \hline \end{array}$$

d) Given the non-canonical maximization L.P.P. below, transfer the data into a maximum table

Maximize: $f(x, y) = x + 3y$, subject to $x + 2y \leq 10,\ 3x + y \geq 15$.

e) Define the zero slack variable in a L.P.P.

f) Given the L.P.P. below; Maximize $f(x_1, x_2) = x_1 + x_2$

subject to $x_1 + 2x_2 \leq 4,\ 3x_1 + x_2 \leq 6$ and $x_1, x_2 \geq 0$

State the dual Canonical minimization L.P.P.

g) State the duality theorem.

h) Define mixed strategy and pure strategy for a column player of a matrix game.

i) Reduce the following table of the matrix game using domination.

$$\begin{bmatrix} 2 & 1 & 4 & 2 \\ 1 & -2 & 1 & 1 \\ -2 & 6 & 3 & -2 \\ 3 & -3 & 5 & 1 \\ 1 & -2 & 2 & 1 \end{bmatrix}$$

j) Define basic feasible solution of a balanced transportation problem.

k) State VAM to find initial basic feasible solution of a balanced transportation problem

l) Define a balanced assignment problem.

m) Define net input flow at a vertex and a source in a capacitated directed net work.

n) Prove that any flow in a capacitated directed net work satisfies $\sum \phi(v_j) = 0$

o) Define a α −path in a capacitated directed net work.

PART – B

UNIT-I

2\. a) An oil company owns two refineries A and B. Refinery A is capable of producing 20 barrels of gasoline and 25 barrels of fuel oil per day. Refinery B is capable of producing 40 barrels of gasoline and 20 barrels of fuel oil per day. The company requires at least 1000 barrels of gasoline and at least 800 barrels of fuel oil. If the cost of operating refinery A is Rs. 300 per day and B is a Rs. 500 per day. Find the number of days each refinery to be operated so as to minimize costs? Solve graphically. **(9)**

b) State the simplex algorithm for maximum basic feasible table. **(9)**

3\. a) Solve: Minimize $g(x, y, z) = -x - y$
subject to $3x + 6y + 2z \leq 6,$
$y + z \geq 1$
$x, y, z \geq 0$ **(9)**

b) Solve the L.P.P. below **(9)**

x_1	x_2	-1	
-1	-2	-3	$= -t_1$
1	1	3	$= -t_2$
1	1	2	$= -t_3$
-2	4	0	$= f$

UNIT-II

4\. a) State the dual simplex algorithm for minimum table of a L.P.P. **(9)**

b) Solve the noncanonical L.P.P. below: Maximize $f(x, y) = x + 3y$ **(9)**
subject to. $x + 2y \leq 10,$
$-3x - y \leq -15$

5\. a) State and prove duality equation. **(9)**

b) Solve the dual Canonical L.P.P. below. **(9)**

	x_1	x_2	-1	
y_1	1	-1	-1	$= -t_1$
y_2	-1	-1	-1	$= -t_2$
-1	1	-2	0	$=f$
	$= s_1$	$= s_2$	$=g$	

UNIT-III

6. a) Solve the dual non Canonical L.P.P. below. **(9)**

$$\begin{array}{c|ccc|c|l}
 & \enclose{circle}{x_1} & \enclose{circle}{x_2} & x_3 & -1 & \\
\hline
\enclose{circle}{y_1} & 1 & -1 & 2 & 1 & = -0 \\
y_2 & 2 & 0 & 2 & -1 & = -t_1 \\
y_3 & 0 & 1 & -1 & -1 & = -t_2 \\
\hline
-1 & 1 & -1 & 3 & 0 & = f \\
\hline
 & =0 & =0 & =s_1 & =g &
\end{array}$$

b) Solve the matrix game below $\begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix}$ **(9)**

7. a) Solve the dual noncanonical L.P.P. below. **(9)**

$$\begin{array}{c|cc|c|l}
 & \enclose{circle}{x_1} & x_2 & -1 & \\
\hline
\enclose{circle}{y_1} & 2 & -1 & -1 & = -0 \\
y_2 & -1 & 1 & -1 & = -t_1 \\
\hline
-1 & 2 & 1 & 0 & = f \\
\hline
 & =0 & =s_1 & =g &
\end{array}$$

b) Solve the matrix game $\begin{bmatrix} \frac{-5}{3} & 0 \\ 5 & \frac{-10}{3} \end{bmatrix}$ **(9)**

UNIT-IV

8. a) State the transformation algorithm to solve a balanced transportation problem. **(9)**

b) Solve the assignment problem below. **(9)**

4	6	5	10
10	9	7	13
7	11	8	13
12	13	12	17

9. a) State the Hungarian algorithm to solve a balanced assignment problem. (9)

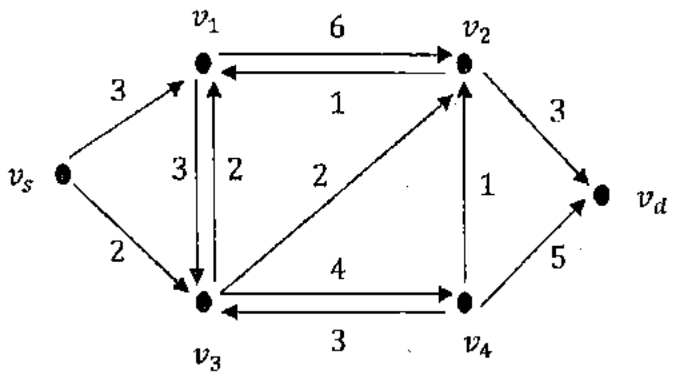
b) Solve the transformation problem below. (9)

5	12	8	50	26
11	4	10	8	20
14	50	1	9	30
15	20	26	15	

UNIT-V

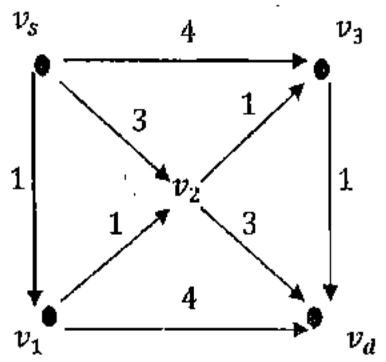
10. a) State the maximal flow algorithm. (9)

b) Solve the shortest path network problem below. (9)



11. a) If N = [V, E] is a capacitated directed network with unique fixed source and unique fixed sink, no edges into the source and no edges out of the sink. Then show that there exists a cut whose capacity is equal to the value of the maximal flow. (9)

b) Solve the shortest path network problem below, also find the shortest path and path value. (9)



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MAT 501.1 Reg. No. ....

CREDIT BASED FIFTH SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2013
MATHEMATICS
PAPER V: ALGEBRA AND DIFFERENTIAL EQUATIONS

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A
3x10=30

1. a) Solve $(D^3 - 7D^2 + 16D - 12)y = 0$

b) Find the particular solution of $(D^2 + 5D + 6)y = e^{-2x}$

c) Find the complementary solution of $(D^2 - 4D - 5)y = 4\,Cos\,3x$

d) Transform $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10x$ into a differential equation with constant coefficients using the substitution $z = log\,x$.

e) Reduce $x\frac{d^2y}{dx^2} + 2\,\frac{dy}{dx} + xy = 0$ to normal form.

f) Verify that $y = x$ is a solution of $x^2\frac{d^2y}{dx^2} - (x^2 + 2x)\frac{dy}{dx} + (x+2)y = x^3e^x$

g) Show that $L\left\{t^{5/2}\right\} = \frac{15}{8s^3}\sqrt{\frac{\pi}{s}}$.

h) Find an inverse transform of $f(s) = \frac{k}{s(s^2+k^2)}$ using convolution theorem.

i) Evaluate $L^{-1}\left\{\frac{15}{s^2+4s+13}\right\}$.

j) Define associate ring.

k) Prove that every field is an integral domain.

l) If F is a field, prove that its only ideals are (0) and F itself.

m) Prove that a Euclidean ring possess a unit element.

n) Let R is a Euclidean ring and suppose that for a, b, c ϵ R, a | bc with g.c.d.(a,b)=1. Prove that a | c.

o) Prove that $x^2 + x + 1$ is irreducible over F, the field of integer mod 2.

PART - B
UNIT-I

2. a) Solve $(D^2 - 4D + 3)y = sin3x\;cos2x$ **(6)**

b) Solve $(D^2 - D - 2)y = 20\,sin2x + 4e^{3x}$ **(6)**

c) Solve $(D^2 - 1)y = 2 + 5x$ **(6)**

3. a) Solve $(D^2+9)y = Cos^3x$ (6)

b) Solve $(D^2-4D-5)y = e^{2x} + 3\,Cos4x$ (6)

c) Solve $(D^3-D^2-D+1)y = 1+x^2$ (6)

UNIT-II

4. a) Solve $(D^3-8)y = xe^x + 3e^{-x}$ (6)

b) Solve $\frac{d^2y}{dx^2} + tan\,x\frac{dy}{dx} + y\,cos^2x = 0$ by the method of changing independent variable. (6)

c) Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - 5y = sin\,(\log x)$ (6)

5. a) Solve $(D^4-1)y = e^x cos\,x$ (6)

b) Solve $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 6x^2+2x+1$ (6)

c) Solve $(D^2+1)y = Sec^2x$ by the method of variation of parameters. (6)

UNIT-III

6. a) If F (t) has a Laplace transform and if $F(t+w) = F(t)$, then prove that (6)

$$L\{F(t)\} = \frac{1}{1-e^{-sw}}\int_0^w e^{-s\beta}F(\beta)d\beta$$

b) Express F (t) in terms of α − function and find (6)

L {F(t)} where $F(t) = \begin{cases} t^2 & 0<t<2 \\ t-1 & 2<t<3 \\ 7 & t>3 \end{cases}$

c) (i) Obtain $L^{-1}\left\{\frac{3s+1}{(s+1)^4}\right\}$ (ii) Obtain $L\{t^2\sin kt\}$ (6)

7. a) Solve $y^{//}(t) - y(t) = 5\sin 2t,\ y(0)=0,\quad y^{/}(0)=1$ (9)

b) Find the Laplace transform of the function

T (t, c) = $\begin{cases} 1 & 0<t<c \\ -1 & c<t<2c \end{cases}$ T (t + 2c, c) = T (t, c) . (9)

UNIT-IV

8. a) Show that the communicative ring D is an integral domain if and only if for a, b, c ϵ D with $a \neq 0$ the relation ab = ac implies that b = c. (6)

b) If R is a communicative ring with unit element, whose only ideals are (o) and R itself, then prove that R is a field. (6)

c) Let $\phi : J\left[\sqrt{2}\right] \rightarrow J\left[\sqrt{2}\right]$ be defined by $\phi\left(a + b\sqrt{2}\right) = a - b\sqrt{2}$. Show that ϕ is an onto homomorphism. Find its kernel. (6)

9. a) If every $x \epsilon R$ satisfies $x^2 = x$ in a ring R, then prove that the ring R is commutative. (6)

b) Prove that a finite integral domain is a field. (6)

c) Define homomorphism of rings. If ϕ is a homomorphism of R into $R^{/}$, then prove that i) ϕ (0) = 0
ii) $\phi(-a) = -\phi(a)$ for every $a \epsilon R$. (6)

UNIT-V

10. a) If R is a ring of integers and p is a prime number, then prove that P = (p) is a prime ideal of R. (6)

b) Let R be a Euclidean ring and a, $b \epsilon$ R. If $b \neq 0$ is not a unit in R, then prove that d(a) < d (ab). (6)

c) If p is prime number of the form 4n + 1, prove that there exist integers a and b such that $p = a^2 + b^2$ (6)

11. a) Prove that an element $'a'$ in a Euclidean ring is a unit if and only if d(a) = d(1). (6)

b) Let R be a commutative ring with unit element and M be an ideal of R. Prove that M is a maximal ideal of R if $\frac{R}{M}$ is a field. (6)

c) If f(x) and g(x) are any two nonzero elements of F[x], then prove that $deg\left(f(x)g(x)\right) = deg\left(f(x)\right) + deg\left(g(x)\right)$ (6)

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MAT 502.1 Reg. No. ....

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MATHEMATICS
PAPER VI: DISCRETE ATHEMATICS

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A

3x10=30

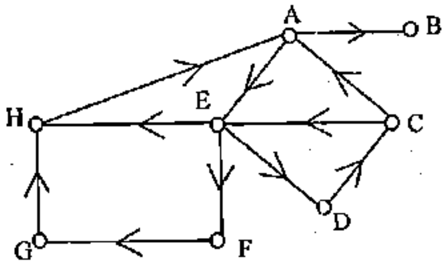
1. a) Prove that $2^n > n^3$ for $n \geq 10$ by induction.

b) If A, B, C are three sets such that $|A| = 2, |B| = 5, |C| = 3, |A \cup B| = 5, |A \cup C| = 4, |B \cup C| = 5$ and $|A \cap B \cap C| = 1$ find $|A \cup B \cup C|$.

c) Define lattice and give an example.

d) In the following graph identify

(i) An elementary circuit
(ii) A simple circuit
(iii) A simple circuit which is not elementary



e) Prove that the number of odd vertices in a graph is always even.

f) Show that a tree with 2 or more vertices has at least 2 leaves.

g) A connected graph G has 5 vertices and 10 edges, and T is a spanning tree of G. Answer the following:
(a) What is the sum of the degrees of all vertices of G.
(b) What is the number of branches and the number of chords, with respect to T.

h) Prove that any circuit and the compliment of any spanning tree in a connected graph must have at least one edge in common.

i) Define binary prefix code and give an example.

j) Analyse the time complexity of the algorithm LARGEST 1.

k) When do we say two states in a finite state machine are 1-equivalent.

l) Show that the input sequence 0010110 is rejected by the finite state machine given below :

States	Inputs		Outputs
	0	1	
A	B	A	0
B	A	C	0
C	B	D	0
D	B	A	1

m) Find the generating function for the numeric function $a_r = 5^{r+2}$.

n) Obtain the characteristic root of the difference equation

$$a_r + 6a_{r-1} + 12a_{r-2} + 8a_{r-3} = 0$$

o) Find a particular solution of the difference equation $a_r - 5a_{r-1} + 6a_{r-2} = 1$

PART - B

UNIT-I

2. a) Prove that the set of all real numbers between 0 and 1 is uncountably infinite. (6)

b) Define conditional probability. Out of 1,00,000 people, 51,500 are female and 48,500 are male. Among the females 9,000 are bald, and among the males 30,200 are bald. Suppose we are going to choose a person at random:

i) Given that the person chosen at random is bald, what is the probability that the person is male?

ii) Given that the person chosen at random is female, what is the probability that the person is not bald? (6)

c) Provide a step-by-step derivation to generate the sentence C = A + D * (D + B) using following productions:

asgn–stat → id = exp
exp → exp + term
exp → term
term → term * factor
term → factor
factor → (exp)
factor → id
id → A
id → B
id → C
id → D (6)

3. a) Show by induction that any integer composed of 3^n identical digits is divisible by 3^n. (6)

b) Let $(P, \leq)$ be a partially ordered set in which the length of the longest chain is n. Show that the elements in P can be partitioned into n disjoint antichains. (6)

c) If A, B, C are three arbitrary sets prove that (A – B) – C = (A – C) – B. (6)

UNIT-II

4. a) Prove that an undirected graph possesses an Eulerian path if and only if it is connected and has either zero or two vertices of odd degree. (6)

   b) If in a graph with n vertices, there is a path from vertex v_1 to a vertex v_2, then show that there is a path of not more than $n - 1$ edges from v_1 to v_2. (6)

   c) Prove that there is always a Hamiltonian path in a directed complete graph. (6)

5. a) Let G be a linear graph with n vertices. If the sum of the degrees for each pair of vertices in G is n-1 or larger, then prove that there exists a Hamiltonian path in G. (6)

   b) For any connected planar graph, show that v-e+r = 2, where v is the number of vertices, e the number of edges and r is the number of regions. (6)

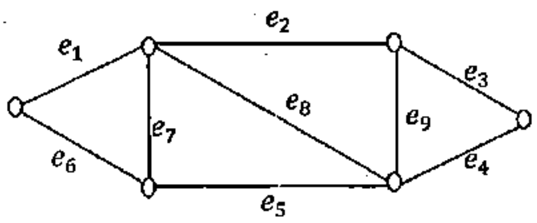
   c) Show that a planar graph on n vertices can have at most $3n - 6$ edges. (6)

UNIT-III

6. a) Prove that in any tree, the number of vertices is one more than the number of edges. (6)

   b) Show that in any connected graph, every circuit has an even number of edges in common with every cutset. (6)

   c) Define fundamental circuit and fundamental cutset. Also determine all the fundamental circuits and all the fundamental cutsets with respect to the spanning tree T = { e_1, e_7, e_5, e_9, e_3} in the following graph: (6)



7. a) Describe an algorithm for determining a minimum spanning tree of a connected weighted graph. (6)

   b) Define a tree. Also show that a circuitless graph with v vertices and v-1 edges is a tree. (6)

   c) In a graph G, let T be a spanning tree and $D = \{e_1, e_2, e_{3,} \ldots, e_k\}$ be a fundamental cutset in which e_1 is a branch and $e_2, e_3, \ldots, e_k$ are chords of T. Show the following:

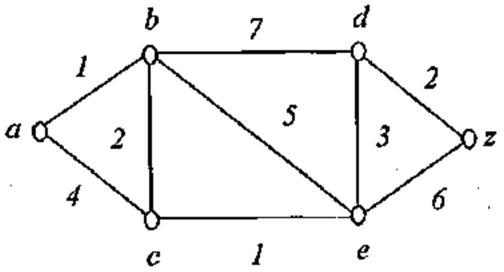
      (i) e_1 is contained in the fundamental circuits corresponding to e_i , i = 2, 3, ... k and

      (ii) e_1 is not contained in any other fundamental circuits. (6)

UNIT-IV

8. a) Show that the language $L = \{a^k \mid k = i^2,\ i \geq 1\}$ is not a finite state language. (9)

   b) Using shortest path algorithm find the shortest distance from a to z in the following graph: (9)



9. a) State the algorithm 'BUBBLESORT' used to sort 'n' numbers stored in n registers. Also analyse its time complexity. (9)

   b) Show that the language $L = \{a^k b^k \mid k \geq 1\}$ is not a finite state language. (9)

UNIT-V

10. a) If $3a_r - 5a_{r-1} + 2a_{r-2} = r^2 + 5$, $a_3 = 3$ and $a_4 = 6$ find a_5 and a_2. (6)

    b) Determine the numeric function corresponding to the generating function $A(z) = \frac{2}{1-4z^2}$ (6)

    c) Find the particular solution of the difference equation: $a_r - 4a_{r-1} + 4a_{r-2} = (r+1)2^r$ (6)

11. a) Find the homogeneous solution of the difference equation. $4a_r - 20a_{r-1} + 17a_{r-2} - 4a_{r-3} = 0$ (6)

    b) If $a_r = 3^r,\ r \geq 0$ and $b_r = 2^r,\ r \geq 0$ and c is the numeric function $c = a * b$, then show that $c_r = 3^{r+1} - 2^{r+1},\ r \geq 0$ (6)

    c) Find the particular solution of the difference equation $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$ (6)

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MAT 502.4 Reg. No. ....

CREDIT BASED FIFTH SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2013
MATHEMATICS
PAPER VI: LINEAR PROGRAMMING

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A

3x10=30

1. a) Define: a) line segment in R^n
b) bounded set in R^n

b) State the canonical minimization LPP represented by

x	y	-1	
1	2	3	$=-t_1$
4	5	6	$=-t_2$
7	8	9	$=f$

c) Pivot on $a_{11} = 2$ in the following canonical maximum table:

x_1	x_2	-1	
2	1	8	$=-t_1$
1	2	10	$=-t_2$
30	50	0	$=f$

d) Given the L.P.P. below,

Maximize: $f(x_1, x_2) = x_1 + 2x_2$

subject to: $x_1 - x_2 \leq 0$

$-x_1 + x_2 \leq -1$

$x_1, \; x_2 \geq 0$

State the dual canonical minimization LPP.

e) Define unconstrained variables in a LPP.

f) Define complementary slackness.

g) Reduce the table of the matrix game $\begin{bmatrix} 2 & -3 & 2 \\ -3 & 4 & -3 \\ 2 & -3 & 6 \end{bmatrix}$ using domination.

h) Define mixed strategy and pure strategy for row player in the matrix game.

i) Let $A = (a_{ij})$ be a $m \times n$ matrix. If the column player chooses pure strategy and row player uses mixed strategy $P = \begin{bmatrix} p_1 \\ p_2 \\ \cdot \\ \cdot \\ \cdot \\ p_m \end{bmatrix}$ then what is the expected value $E_j(P)$ of his winnings?

j) State the balanced assignment problem.

k) Find all permutation set of zeros in the following table of balanced assignment problem:

0	0	0	0
4	1	0	1
0	2	0	0
1	0	0	0

l) Convert the following unbalanced transportation problem into a balanced transportation problem:

3	2	1	30
2	5	9	75
40	30	50	

m) State maximal flow network problem.

n) Define a cut, cutset and capacity of a cut in a directed network.

o) Prove that any flow in a capacitated directed network satisfies $\sum_j \phi(V_j) = 0$

PART - B

UNIT-I

2\. a) An oil company owns two refineries, say refinery A and refinery B. Refinery A is capable of producing 20 barrels of gasoline and 25 barrels of fuel oil per day. Refinery B is capable of producing 40 barrels of gasoline and 20 barrels of fuel oil per day. The company requires at least 1000 barrels of gasoline and at least 800 barrels of fuel oil. If it costs \$ 300 per day to operate refinery A and \$ 500 per day to operate refinery B, how many days should each refinery be operated by the company so as to minimize costs? Solve graphically. (9)

b) Apply simplex algorithm to the following maximum table:

$$\begin{array}{|cc|c|l} \multicolumn{1}{c}{x_1} & \multicolumn{1}{c}{x_2} & \multicolumn{1}{c}{-1} & \\ \hline -1 & -2 & -3 & = -t_1 \\ 1 & 1 & 3 & = -t_2 \\ 1 & 1 & 2 & = -t_3 \\ \hline -2 & 4 & 0 & = f \\ \hline \end{array}$$

(9)

3. a) State the complete simplex algorithm for maximum table. (9)

b) Apply simplex algorithm to the following maximum table:

$$\begin{array}{|cc|c|l} \multicolumn{1}{c}{x_1} & \multicolumn{1}{c}{x_2} & \multicolumn{1}{c}{-1} & \\ \hline -1 & 1 & 1 & = -t_1 \\ 1 & -1 & 3 & = -t_2 \\ \hline 1 & 2 & 0 & = f \\ \hline \end{array}$$

(9)

UNIT-II

4. a) Solve the non-canonical LPP below:

Maximize $f(x,y,z) = x + 2y + z$

subject to: $x + y + z = 6$

$x + y \leq 1$

$x, z \geq 0$

(9)

b) For any pair of feasible solutions of dual canonical LPPs prove that

$$g - f = SX^{/} + Y^{/}T$$

(9)

5. a) State the dual simplex algorithm for minimum tableau of a LPP. (9)

b) Solve the noncanonical LPP below:

Maximize $f(x,y) = x + 3y$ (9)

subject to: $x + 2y \leq 10,$

$-3x - y \leq -15$

UNIT-III

6. a) Solve the dual non Canonical L.P.P. below. (9)

$$\begin{array}{c|ccc|c|l}
 & \textcircled{x_1} & \textcircled{x_2} & x_3 & -1 & \\
\hline
\textcircled{y_1} & 1 & -1 & 2 & 1 & = -0 \\
y_2 & 2 & 0 & 2 & -1 & = -t_1 \\
Y_3 & 0 & 1 & -1 & -1 & = -t_2 \\
\hline
-1 & 1 & -1 & 3 & 0 & = f \\
\hline
 & =0 & =0 & =s_1 & =g &
\end{array}$$

b) Find the optimal strategies for the row and column players and the Von-Neumann value of the matrix game with pay off matrix

$$\begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix}$$ (9)

7. a) Solve the dual Canonical L.P.P. below: (9)

$$\begin{array}{c|cc|c|l}
 & \textcircled{x_1} & x_2 & -1 & \\
\hline
\textcircled{y_1} & 2 & -1 & -1 & = -0 \\
y_2 & -1 & 1 & -1 & = -t_1 \\
\hline
-1 & 2 & 1 & 0 & = f \\
\hline
 & =0 & =s_2 & =g &
\end{array}$$

b) Find the optimal strategies for the row and column players and the Von-Newmann value of the matrix game with pay off matrix $\begin{bmatrix} \frac{-5}{3} & 0 \\ 5 & \frac{-10}{3} \end{bmatrix}$ (9)

UNIT-IV

8. a) State the transformation algorithm to solve a balanced transportation problem. (9)

b) Solve the following assignment problem using Hungarian algorithm. (9)

2	3	2	4
5	8	4	3
5	9	5	2
7	6	7	4

9. a) State the Hungarian algorithm to solve a balanced assignment problem. (9)

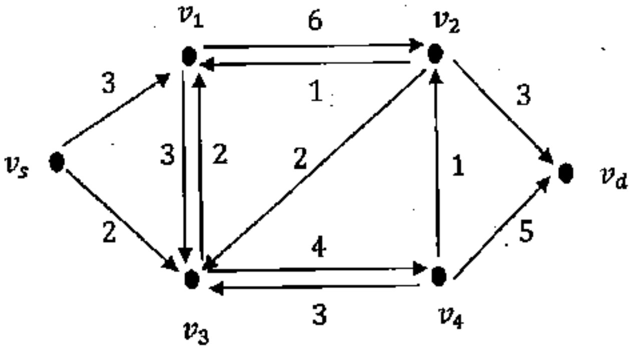
b) Solve the transformation problem below. (9)

7	2	4	10
10	5	9	20
7	3	5	30
20	10	30	

UNIT-V

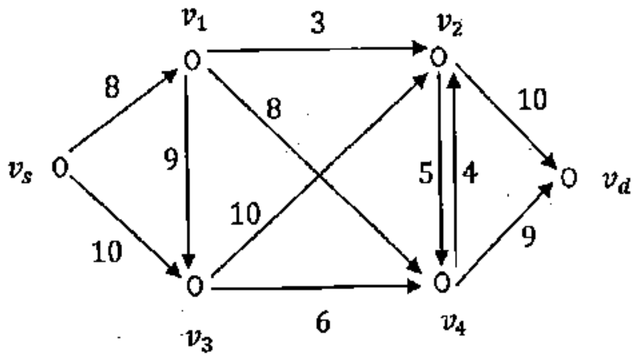
10. a) Show that in a capacitated directed network with unique fixed source and unique fixed sink, no edges into the source and no edges out of the sink, the value of the maximum flow is less than or equal to the minimal cut capacity. (9)

b) Solve the shortest path network problem below by using Dijkstras Algorithm. Also give the shortest path and the path value. (9)



11. a) State the shortest path algorithm – I. (9)

b) Solve the maximal flow network problem below. Display the corresponding cut and cutset. (9)



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MAT 501.1 **Reg. No. ....**

CREDIT BASED FIFTH SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2014

MATHEMATICS

PAPER V: DIFFERENTIAL EQUATIONS AND RING THEORY

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A

3x10=30

1\. a) Solve $\left(D^3-3D^2+3D-1\right)y=0$

b) Find the particular integral of $\left(D^2-6D+13\right)y=5e^{2x}$

c) Find the complementary solution of $\left(D^2-D-2\right)y=20\sin 2x$

d) Transform $(x+1)^2y_2-3(x+1)y_1+4y=x^2$ into differential equation with constant coefficients by putting $z=x+1$ and $u=\log z$

e) Find A in the method of variation of parameters to solve $\left(D^2+1\right)y=\sec x\tan x$

f) Reduce $\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+x^2y=0$ to normal form.

g) Find $L\{H(t)\}$ where $H(t)=\begin{cases}4 & 0<t<1\\ 3 & t>1\end{cases}$

h) Evaluate $L\left\{\cos^2 kt\right\}$

i) Derive $L\left\{t^n\right\}$ for any positive integer n.

j) Define (i) Field (ii) Integral domain

k) If U is an ideal of R and $1\in U$ then show that $U=R$.

l) If p is a prime number then prove that J_p, the ring of integers modulo p is a field.

m) If $a\,|\,b$ and $b\,|\,a$ for a, b in an integral domain R with unit element, then prove that $a=ub$ where u is a unit in R.

n) Prove that x^2+x+1 is irreducible over F where F is the field of integers modulo 2.

o) Find all the units in $J[i]$

PART - B

UNIT-I

2. a) Solve $\left(D^3-12D+16\right)y=\left(e^x+e^{-2x}\right)^2$ (6)

b) Solve $\left(3D^2+D-14\right)y=8e^{2x}+\cos 5x$ (6)

c) Solve $\left(D^4+D^3+D^2\right)y=5x^2$ (6)

3. a) Solve $\left(D^2-8D+9\right)y=8\sin 5x$ (6)

b) Solve $\left(D^2+9\right)y=\cos^3 x$ (6)

c) Solve $\left(D^3-D^2-D+1\right)y=1+x^2$ (6)

UNIT-II

4. a) Solve $\left(D^2+2D+5\right)y=xe^x$ (6)

b) Solve $x^3\dfrac{d^3y}{dx^3}+2x^2\dfrac{d^2y}{dx^2}+2y=10x$ (6)

c) Solve $x\dfrac{d^2y}{dx^2}+2(4x-1)\dfrac{dy}{dx}-(9x-2)y=x^3e^x$ by reduction of order. (6)

5. a) Solve $x^2\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}-5y=\sin(\log x)$ (6)

b) Solve $x^2\dfrac{d^2y}{dx^2}-2\left(x^2+x\right)\dfrac{dy}{dx}+\left(x^2+2x+2\right)y=0$ by reduction to normal form. (6)

c) Solve $\left(1+x^2\right)y_2+xy_1+2y=0$ by changing the independent variable. (6)

UNIT-III

6. a) If $F(t)$ is a periodic function with period w, then derive the formula for

$L\{F(t)\}$ (6)

b) Express F (t) in terms of α – function and find (6)

$L\{F(t)\}$ where $F(t)=\begin{cases} t^2 & 0<t<2 \\ t-1 & 2<t<3 \\ 7 & t>3 \end{cases}$

c) Evaluate (i) $L^{-1}\left\{\dfrac{s-5}{s^2+6s+13}\right\}$ (ii) $L\left\{t^2\sin kt\right\}$ (6)

7. a) Find function $g(t)$ for which $g(t)=L^{-1}\left\{\frac{3}{s}-\frac{4e^{-s}}{s^2}+\frac{4e^{-3s}}{s^2}\right\}$ **(6)**

b) Solve $y''(t)+6y'(t)+9y(t)=6t^2e^{-3t}, \quad y(0)=0, \quad y'(0)=0$ **(6)**

c) Find the transform of the function

$$\psi(t,c)=\begin{cases}1 & 0<t<c\\ 0 & c<t<2c\end{cases}$$

$\psi(t+2c,c)=\psi(t,c)$ **(6)**

UNIT-IV

8. a) Prove that a finite integral domain is a field. **(6)**

b) Define Kernel of a homomorphism. If ϕ is a homomorphism of a ring R into a ring R^1, then prove that kernel of ϕ is an ideal of R. **(6)**

c) If U is an ideal of a ring R, then prove that $r(U)=\{r\in R \mid xu=0, \forall u\in U\}$ is also an ideal of R. **(6)**

9. a) Define zero divisor. If R is a ring then for $a,b\in R$, prove that
(i) $a\cdot 0=0\cdot a=0$ (ii) $a(-b)=(-a)b=-(ab)$ **(6)**

b) Let $\phi : J\left(\sqrt{2}\right)\rightarrow J\left(\sqrt{2}\right)$ be defined by $\phi\left(a+b\sqrt{2}\right)=a-b\sqrt{2}$. Show that ϕ is an onto homomorphism. Find its Kernel. **(6)**

c) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Prove that R is a field. **(6)**

UNIT-V

10. a) Let R be a Euclidean ring. Prove that any two elements a and b in R have a greatest common divisor 'd' which can be expressed as $d=\lambda a+\mu b$ for some λ, μ in R. **(6)**

b) If p is a prime number of the form $4n+1$ then prove that the congruence relation $x^2\equiv -1(\text{mod } p)$ has a solution. **(6)**

c) If $f(x)$ and $g(x)$ are two non-zero elements of polynomial ring $F(x)$, then prove that $\deg\left(f(x)g(x)\right)=\deg f(x)+\deg g(x)$ **(6)**

11. a) Define maximal ideal of a ring R. Let R be a commutative ring with unit element and M be an ideal of R. Prove that M is a maximal ideal of R if and only if R/M is a field. **(6)**

b) Prove that every element in a Euclidean ring R is either a unit in R or can be written as the product of a finite number of prime elements of R. **(6)**

c) Let R be a Euclidean ring and let A be an ideal of R. Prove that there exists an element a_0 in A such that A consists exactly of all a_0x as x ranges over R **(6)**

MAT 502.1 **Reg. No. ....**

CREDIT BASED FIFTH SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2014
MATHEMATICS
PAPER VI: DISCRETE ATHEMATICS

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A

3x10=30

1. a) Prove that $2^n > n^3$ for $n \geq 10$ by induction.

b) If $A = \{p, q, r\}$, find $|P(A) \times A|$

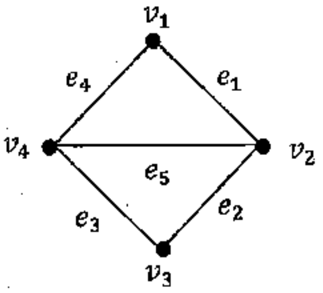
c) Define partially ordered set and give an example.

d) Define '2-factor' of a graph with an example.

e) Prove that the number of odd vertices in a graph is always even.

f) What is the difference between an Eulerian path and a Hamiltonian path.

g) List any three cutsets for the following graph.



h) Prove that there is one and only one path between any two vertices in a tree.

i) Obtain a binary tree for the following prefix code $\{1, 01, 000, 001\}$.

j) Represent the model of 'modulo 3 sum counter' graphically.

k) Analyse the time complexity of the algorithm 'LARGEST1'.

l) When is a problem said to be tractable? Give an example for a tractable problem.

m) Define forward and backward differences of a numeric function.

n) Determine the numeric function corresponding to the generating function

$$A(z) = \frac{2}{1-4z^2}.$$

o) Find a particular solution of the difference equation $a_r - 5a_{r-1} + 6a_{r-2} = 1$

PART - B

UNIT-I

2. a) Prove that the set of real numbers between 0 and 1 is not a countably infinite set. **(6)**

   b) Determine the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7. **(6)**

   c) Provide a step-by-step derivation to obtain the string -010 using the following set of productions:

   signed integer → sign integer
   sign → +
   sign → –
   integer → digit integer
   integer → digit
   digit → 0
   digit → 1 **(6)**

3. a) If the length of the longest chain in a partially ordered set $(P, \le)$ is n, then show that the elements in P can be partitioned into n disjoint antichains. **(6)**

   b) For the sets $A_1, A_2, ..., A_r$ prove that

   $$|A_1 \cup A_2 \cup .....\cup A_r| = \sum_i |A_i| - \sum_{1\le i<j\le r} |A_i \cap A_j| + \sum_{1\le i<j<k\le r} |A_i \cap A_j \cap A_k| +$$

   $$...+(-1)^{r-1}|A_1 \cap A_2 \cap A_3 \cap ....\cap A_r|$$ **(6)**

   c) In how many ways can a group of 8 people be divided into committees, subject to the constraint that each person must belong to exactly one committee and each committee must contain at least two people? **(6)**

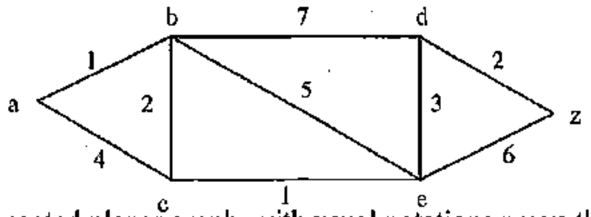
UNIT-II

4. a) Prove that an undirected graph possesses an Eulerain path if and only if it is connected and has either zero or two vertices of odd degree. **(6)**

   b) If in a graph with n vertices, there is a path from vertex v_1 to v_2 then show that there is a path of not more than $n-1$ edges from v_1 to v_2 **(6)**

   c) Prove that there is always a Hamiltonian path in a directed complete graph. **(6)**

5. a) Find the shortest distance from vertex 'a' to vertex 'z' in the graph given below. **(6)**



   b) For any connected planar graph, with usual notations prove that $v\text{-}e+r = 2$. **(6)**

   c) Let G be a linear graph on with n vertices. If the sum of degrees for each pair of vertices in G is n-1 or larger, then prove that there exists a Hamiltonian path in G. **(6)**

UNIT-III

6. a) Define tree. Prove that the number of vertices is one more than the number of edges in a tree. (6)

b) In a graph G, let T be a spanning tree and $D=\{e_1,e_2,...,e_k\}$ be a fundamental cutest in which e_1 is a branch and $e_2,e_3,...,e_k$ are chords of T. Show that

(i) e_1 is contained in the fundamental circuits corresponding to e_i for i=2,3,…,k

(ii) e_1 is not contained in any other fundamental circuits. (6)

c) Construct a prefix code for the following alphabet, given the respective occurrences. (6)

letter	a	b	j	k	v	x	z
Occurrences	82	14	1	4	9	2	1

7. a) When is a set of sequences said to be a prefix code? Give examples one each for

(i) a set which is not a prefix code.

(ii) a binary prefix code having 8 elements. (6)

b) Show that in a connected graph every circuit has an even number of edges in common with every cutset. (6)

c) Define the following with examples.

(i) Fundamental circuit

(ii) Fundamental cutset (6)

UNIT-IV

8. a) Show that the language $L = \{a^k \mid k = i^2,\ i \geq 1\}$ is not a finite state language. (9)

b) Write the algorithm 'BUBBLESORT' used to sort the 'n' numbers stored in n registers. Also justify it with a formal proof. (9)

9. a) Let L be a finite state language accepted by a finite state machine with N states. For any sequence α whose length is N or larger in the language, show that α can be written as uvw such that v is nonempty and uv^iw is also in the language for $i \geq 0$ where v^i is the concatenation of i copies of the sequence v. (9)

b) Show that the language $L = \{a^k b^k \mid k \geq 1\}$ is not a finite state language. (9)

UNIT-V

10. a) Find the numeric function corresponding to the generating function (6)

$$A(z)=\frac{4+z-6z^2}{\left(1-4z^2\right)\left(1+z\right)}$$

b) If $a=\alpha_0+\alpha_1 r+\alpha_2 r^2 ......+\alpha_n r^n$ show that a is $0(r^n)$. (6)

c) Find the homogeneous solution of the difference equation. (6)

$$4a_r - 20a_{r-1} + 17a_{r-2} - 4a_{r-3} = 0$$

11. a) Obtain the particular solution of the difference equation (6)

$$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$$

b) If $a_r = 5^r, r \geq 0$ and $b_r = 3^r, r \geq 0$ determine C_r such that $c=a*b$ (6)

c) Write the recurrence relation for the Fibonacci sequence of numbers and find its homogenous solution. (6)

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MAT 502.4 Reg. No. ....

CREDIT BASED FIFTH SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2014
MATHEMATICS
PAPER VI: LINEAR PROGRAMMING

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A

3x10=30

1. a) Define a convex set in R^n and give an example of a convex set in R^2

b) Define canonical maximization LPP.

c) Pivot on $a_{11} = 3$ in the following canonical maximization table.

x_1	x_2	-1	
3	2	1	$= -t_1$
6	1	3	$= -t_2$
9	2	0	= f

d) Define unconstrained variables in a L.P.P.

e) Write the matrix reformulation of canonical maximization L.P.P.

f) Define negative transpose of the minimum table.

g) State the duality theorem.

h) Let $A = (a_{ij})$ be an $m \times n$ matrix game. If the column player chooses pure strategy and row player uses mixed strategy $P = \begin{bmatrix} p_1 \\ p_2 \\ \cdot \\ \cdot \\ \cdot \\ p_m \end{bmatrix}$

then what is the expected value of row players winnings.

i) Reduce the following table of the matrix game using domination.

$$\begin{bmatrix} -1 & 1 & -1 & 2 \\ -1 & -1 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

j) State the balanced assignment problem.

k) Find all permutation sets of zeros in the following table of balanced assignment problem.

0	0	1
0	1	0
2	0	0

l) Define cycle in a table of a balanced transportation problem.

m) Prove that any flow in a capacitated directed network satisfies $\sum_j \phi(v_j) = 0$

n) Define a cut, cutset and capacity of a cut in a directed network.

o) Define an α – path in a capacitated directed network.

PART - B

UNIT-I

2. a) A publishing firm prints two magazines, the Monitor and the Recorder, each in units of one hundred. Each unit of the Monitor requires 1 unit of ink, 3 units of paper, and 4 hours of printing press time to print, each unit of the Recorder requires 2 units of ink, 3 units of paper, and 5 hours of printing press time to print. The firm has 20 units of ink, 40 units of paper, and 60 hours of printing press time available. If the profit realized upon sale in $200 per unit of the Monitor and $300 per unit of the Recorder, how many units of each magazine should the firm print so as to maximize profits? Solve graphically. (9)

b) Solve Maximize: $f(x,y) = x$

subject to $x + y \leq 1$

$x - y \geq 1$

$y - 2x \geq 1$

$x, y \geq 0$ (9)

3. a) State the complete simplex algorithm for maximum table. (9)

b) Apply simplex algorithm to the following maximum table

$$\begin{array}{cc|c|l} x & y & -1 & \\ \hline 2 & 1 & 8 & = -t_1 \\ 1 & 2 & 10 & = -t_2 \\ \hline 30 & 50 & 0 & = f \end{array}$$

(9)

UNIT-II

4. a) Solve the non-canonical LPP below:

Maximize $f(x,y) = x + 3y$

subject to $x + 2y \leq 10,$

$3x + y \leq 15$ (9)

b) State and prove duality equation. (9)

5. a) Solve the non-canonical LPP below:

Maximize $f(x,y,z) = 2x + y - 2z$ (9)

subject to $x + y + z \leq 1$

$y + 4z = 2$

$x, y, z \geq 0$

b) Solve the dual Canonical L.P.P. below. (9)

$$\begin{array}{c|cc|c|l} & x_1 & x_2 & -1 & \\ \hline y_1 & -2 & 1 & -2 & = -t_1 \\ y_2 & 1 & -1 & -1 & = -t_2 \\ \hline -1 & 1 & 1 & 0 & = f \\ \hline & = s_1 & = s_2 & = g & \end{array}$$

UNIT-III

6. a) Solve the dual non Canonical L.P.P. (9)

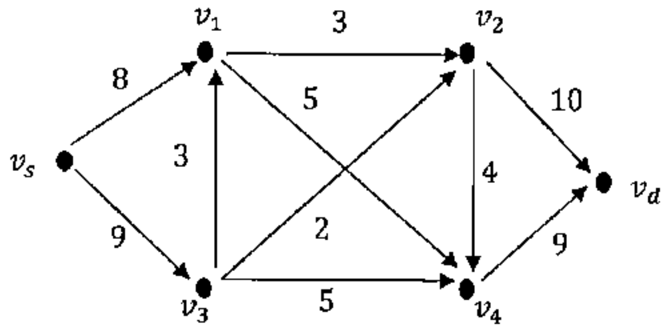
$$\begin{array}{c|cc|c|l} & \enclose{circle}{x_1} & x_2 & -1 & \\ \hline \enclose{circle}{y_1} & 1 & -1 & -2 & = -0 \\ y_2 & -2 & 2 & -1 & = -t_1 \\ \hline -1 & 0 & 1 & 0 & = f \\ \hline & = 0 & = s_2 & = g & \end{array}$$

b) Solve the matrix game $\begin{bmatrix} \frac{-5}{3} & 0 \\ 5 & \frac{-10}{3} \end{bmatrix}$ (9)

UNIT-V

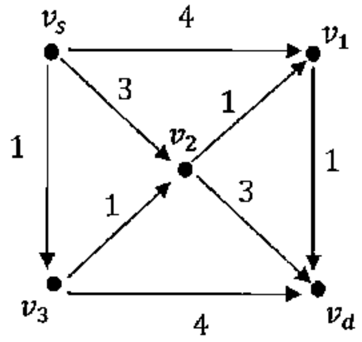
10. a) If $N = [V, E]$ is a capacitated directed network with unique fixed source and unique fixed sink, no edges into the source and no edges out of the sink. Then show that there exists a cut whose capacity is equal to the value of the maximal flow. **(9)**

b) Solve the maximal flow network problem below. Display the corresponding minimal cut and cutset. **(9)**



11. a) State the maximal flow algorithm. **(9)**

b) Solve the shortest path network problem below. Also find the shortest path and path value. **(9)**



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MAT 501.1 Reg. No. ....

CREDIT BASED FIFTH SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2015

MATHEMATICS

PAPER V: DIFFERENTIAL EQUATIONS AND RING THEORY

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A

3x10=30

1. a) Find the complementary function of $\left(D^3-3D^2+4D-2\right)y=e^x+\cos x$

b) Find the particular integral of $\left(D^2+D+1\right)y=\sin 2x$

c) Solve $\left(D^2-5D+6\right)y=e^{4x}$

d) Transform the equation $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-5y=\sin(\log_e x)$ into a differential equation with constant coefficients using the substitution $z=\log_e x$.

e) Reduce the equation $y_2-4xy_1+(4x^2-3)y=e^{x^2}$ to the normal form.

f) Verify that $y=e^x$ is solution of the equation, $x\frac{d^2y}{dx^2}+2(4x-1)\frac{dy}{dx}-(9x-2)y=x^3e^x$

g) Find $L\left\{\, e^{kt}\,\right\}$ by using the definition.

h) Find $L\left\{t^2 sink\, t\right\}$.

i) Evaluate $L^{-1}\left\{\frac{1}{s(s-2)}\right\}$

j) If p is a prime number, then prove that J_p, the ring of integers mod p is a field.

k) If U is an ideal in a ring R and $1\,\varepsilon\, U$ then prove that U = R.

l) If every $x\,\varepsilon\, R$ satisfies $x^2=x$ then prove that R is commutative.

m) Let R = Z and P = pZ where p is a prime number. Prove that P is a prime ideal.

n) Prove that every Euclidean domain has a unit element.

o) Prove that x^2+1 in irreducible over the integers mod 7.

PART - B

UNIT-I

2. a) Solve $\left(D^2+5D+6\right)y=e^{-2x}+\sin 4x$ (6)

b) Solve $\left(D^2-4D+3\right)y=\sin 3x\cos 2x$ (6)

c) Solve $\left(D^2-1\right)y=2+5x$ (6)

3. a) Solve $\left(D^3-12D+16\right)y=\left(e^x+e^{-2x}\right)^2$ (6)

b) Solve $\left(D^3+D^2+D+1\right)y=2x^3+3x^2-4x+5$ (6)

c) Solve $\left(D^2+D+1\right)y=\sin 2x$ (6)

UNIT-II

4. a) Solve $\left(D^2-2D+2\right)y=e^x\cos x$ (6)

b) Solve $\dfrac{d^2y}{dx^2}+\tan x\dfrac{dy}{dx}+\cos^2 x.\ y=0$ by changing the independent variable. (6)

c) Solve $\left(D^2+1\right)y=\sec x\tan x$ by the variation of parameters method. (6)

5. a) Solve $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+2y=x^2$ (6)

b) Solve $x\dfrac{d^2y}{dx^2}-(2x-1)\dfrac{dy}{dx}+\left(x-1\right)y=e^x$ by reduction of order method. (6)

c) Solve $x^2\dfrac{d^2y}{dx^2}-2\left(x^2+x\right)\dfrac{dy}{dx}+\left(x^2+2x+2\right)y=0$ by reducing to normal form. (6)

UNIT-III

6. a) Obtain $L\{\sin kt\}$ by using the definition. (6)

b) If $F(t)$ is periodic with period w, then prove that $L\{F(t)\}=\dfrac{1}{1-e^{-sw}}\int_0^w e^{-s\beta}F(\beta)d\beta$

(6)

c) Write F (t) in terms of $\alpha -$ function and find (6)

$$L\{F(t)\} \text{ if } F(t)=\begin{cases} t^2 & 0<t<2 \\ 4 & 2\le t\le 4 \\ 0 & t>4 \end{cases}$$

7. a) Evaluate (i) $L^{-1}\left\{\dfrac{s}{s^2+2s+5}\right\}$ (ii) $L^{-1}\left\{\dfrac{e^{-4s}}{(s+2)^3}\right\}$ (6)

b) Find the Laplace transform of the function $\varphi(t)$ defined by $\varphi(t)=\begin{cases}1 & 0<t<c\\ 0 & c<t<2c\end{cases}$

and $\varphi(t+2c)=\varphi(t)$ for all t. (6)

c) Solve the differential equation, $x''(t)+2x'(t)=8t,\ x(0)=0,\ x'(0)=0$ by the Laplace transform method. (6)

UNIT-IV

8. a) Prove that a finite integral domain is a field. (6)

b) Let $J\left[\sqrt{2}\right]$ be the ring of all real numbers of the form $m+n\sqrt{2},$ m, n are integers. Define $\phi : J\left[\sqrt{2}\right]\rightarrow J\left[\sqrt{2}\right]$ by $\phi\left(m+n\sqrt{2}\right)=m-n\sqrt{2}$. Prove that ϕ is a homomorphism and find its kernel. (6)

c) Let U be an ideal of a ring R and let $r(U)=\{x\,\varepsilon\, R\mid xu=0,\ \forall u\,\varepsilon\, U\}$
Prove that $r(U)$ is an ideal in R. (6)

9. a) Prove that a field is an integral domain. Is the converse true? Justify your answer. (6)

b) Let U, V be ideals of R and UV be the set of all elements that can be written as finite sums of elements uv, where $u\,\varepsilon\, U$ and $v\,\varepsilon\, V$. Prove that UV is an ideal in R. (6)

c) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Prove that R is a field. (6)

UNIT-V

10. a) Let R be a Euclidean ring and let A be an ideal of R. Prove that there exists an element $a_0\,\varepsilon\, A$ such that A consists exactly of all $a_0\,x$ where x ranges over R. (6)

b) In a commutative ring R with unit element, define the relation 'is an associate of' and show that it is an equivalence relation. (6)

c) If p is a prime number of the form $4n+1$, then prove that the congruence equation $x^2\equiv -1$ (mod p) has a solution. (6)

11. a) Let R be a Euclidean ring and $a,b\,\varepsilon\, R$. If $b\neq 0$ is not a unit in R, then prove that $d(a)<d(ab)$ (6)

b) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field. (6)

c) If $f(x)$ and $g(x)$ are any two non-zero elements of $F\left[x\right]$ then prove that, $\deg\left(f(x)g(x)\right)=\deg f(x)+\deg g(x)$ (6)

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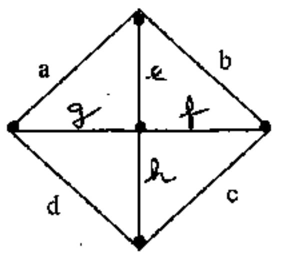
(35)

c) Define a fundamental circuit and list all the fundamental circuits with respect to the spanning tree T = {a, b, c, h} of the following graph. (6)

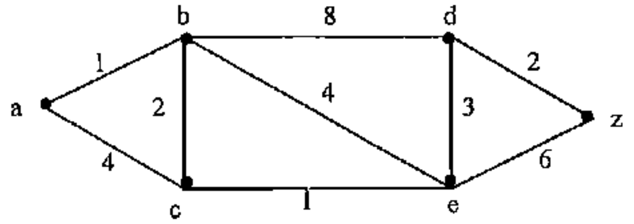


UNIT-IV

8. a) Show that the language $L = \{a^k b^k | k \geq 1\}$ is not a finite state language. (9)

b) Describe the algorithm LARGEST 2 for finding the largest of n numbers. Justify it with a formal proof. (9)

9. a) Using shortest path algorithm find the shortest distance from ‘a’ to ‘z’ in the following graph. (9)



b) Describe the algorithm ‘BUBBLESORT’ used to sort the ‘n’ numbers stored in n registers. Also analyse its time complexity. (9)

UNIT-V

10. a) Write the recurrence relation for the Fibonacci sequence of numbers and find its homogeneous solution. (6)

b) Determine the numeric function a_r, corresponding to the generating function

$$A(Z) = \frac{4+Z-6Z^2}{(1-4Z^2)(1+Z)}$$ (6)

c) Find the particular solution of the difference equation (6)

$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)2^r$$

11. a) Obtain the particular solution for the difference equation (6)

$$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1$$

b) Compute a_0 and a_7 given .

$$3a_r - 5a_{r-1} + 2a_{r-2} = r^2 + 5, a_3 = 3 \text{ and } a_4 = 6$$ (6)

c) Find the particular solution of the difference equation

$$a_r + a_{r-1} = 3r2^r$$ (6)

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MAT 502.1 **Reg. No. ....**

CREDIT BASED FIFTH SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2015
MATHEMATICS
PAPER VI: DISCRETE ATHEMATICS

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A

3x10=30

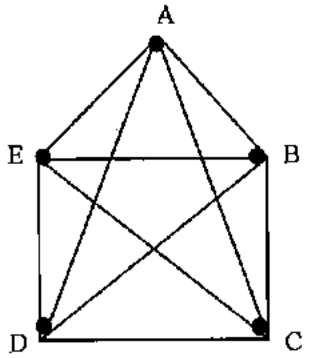
1. a) If $A = \{p,q,r\}$, find $|p(A) \times A|$

b) If the probabilities of A and B are 0.392 and 0.515 respectively and the probability of $A \cap B$ is 0.090 find $p(A \cup B)$ and $p(^{A}|_{B})$.

c) Define partially ordered set, chain and antichain.

d) Define k-factor of a graph with an example.

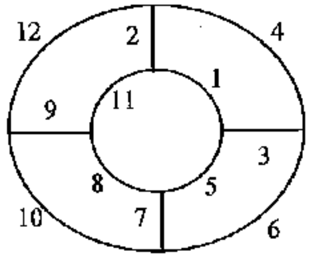
e) Using Euler’s formula show that the following graph cannot be planar.



f) Prove that there is one and only one path between any two vertices in a tree.

g) Obtain a binary tree for the following prefix code {1, 01, 000, 001}.

h) Draw a minimum spanning tree for the following graph



i) Define the terms Eulerian path, Hamiltonian path with one example each.

j) Find the total number of comparisons used in the algorithm BUBBLESORT.

k) Obtain the finite state machine for the modulo 3 sum of the digits {0, 1, 2} in the input signals.

l) Find the output sequence produced for the input sequence 1212 by the following finite state machine.

State	Inputs		Outputs
	1	2	
A	B	C	0
B	C	D	0
C	D	E	0
D	E	B	0
E	B	C	1

m) If $A(Z)=\dfrac{2}{1-2Z}$ find a_r when $r=1$

n) If $a=\alpha_0+\alpha_1 r+\alpha_2 r^2+.....+\alpha_n r^n$ show that a is $0(r^n)$.

o) Find the particular solution of the difference equation $a_r = a_{r-1}+7$

PART - B

UNIT-I

2. a) Provide a step-by-step derivation to generate the sentence $A=(C*D)+B$ using following productions. (6)

asgn – stat → id = exp
exp → exp + term
exp → term
term → term * factor
term → factor
factor → (exp)
factor → id
id → A
id → B
id → C
id → D

b) In how many ways can a group of 8 people be divided into committees, subjected to the constraint that each person must belong to exactly one committee and each committee must contain at least two people. (6)

c) Show that any integer composed of 3^n identical digits is divisible by 3^n. (6)

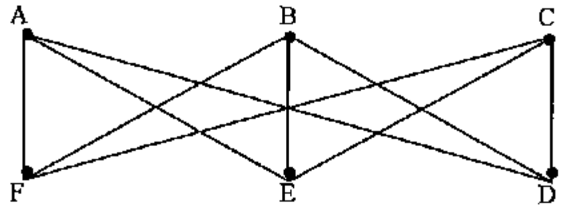
3. a) Determine the number of digits between 1 and 250 that are divisible by anyone of the digits 2, 3, 5 and 7. (6)

b) Let $(P,\leq)$ be a partially ordered set in which the longest chain is 'n'. Show that the elements in P can be partitioned into n disjoint antichains. (6)

c) If A, B, C are any three finite sets show that $(A-B)-C=A-(B\cup C)$. (6)

UNIT-II

4. a) Show that the graph given below is not planar. (6)

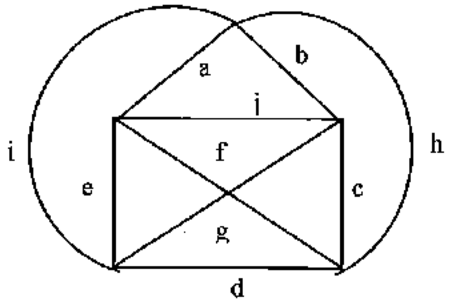


b) Prove that an undirected graph possesses an Eulerian path if and only if it is connected and has either zero or two vertices of odd degree. (6)

c) If in a graph with n vertices there is a path from v_1 to v_2, then show that there is a path of not more than n – 1 edges from v_1 to v_2. (6)

5. a) Prove that there exists a Hamiltonian path in any directed complete graph. (6)

b) Trace an Eulerian circuit and a Hamiltonian circuit in the following graph. (6)



c) With usual notations prove that $v-e+r=2$ for any connected planar graph. (6)

UNIT-III

6. a) Show that the number of vertices is one more than the number of edges in a tree. (6)

b) When is a set of sequences said to be a prefix code. Give examples one each for the following:
(i) a set which can't be a prefix code.
(ii) a set which is a binary prefix code having 8 elements. (6)

c) Prove that in any graph every circuit has an even number of edges in common with every cutset. (6)

7. a) Prove that a graph in which the number of edges is one less than the number of vertices and has no circuit is a tree. (6)

b) Construct a prefix code for the following alphabet given the respective occurrences (6)

Letter	a	b	c	d	e
No. of Occurrences	12	3	6	4	5

(35) 14/10

UNIT-IV

8. a) State the transportation algorithm. (9)

b) Solve the assignment problem below. (9)

8	7	10
7	7	8
8	5	7

9. a) State the Hungarian algorithm. (9)

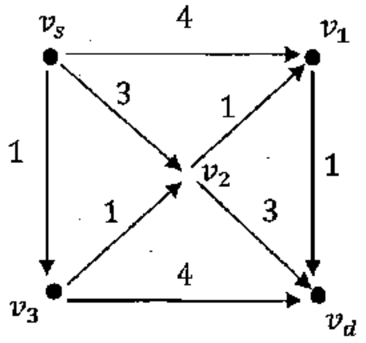
b) Solve the transportation problem below. (9)

7	2	4	10
10	5	9	20
7	3	5	30
20	10	30	

UNIT-V

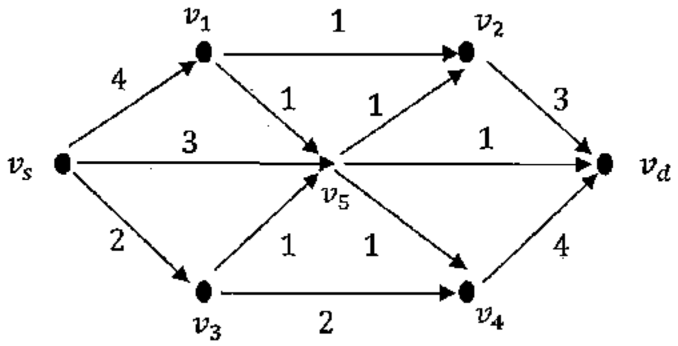
10. a) Show that in a capacitated directed network with unique fixed source and unique fixed sink, no edges into the source and no edges out of the sink, the value of the maximum flow is less than or equal to the minimal cut capacity. (9)

b) Solve the shortest path network problem (9)



11. a) State the shortest path algorithm . (9)

b) Solve the maximal flow network problem (9)



MAT 502.4 Reg. No. ....

CREDIT BASED FIFTH SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2015
MATHEMATICS
PAPER VI: LINEAR PROGRAMMING

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A 3x10=30

1. a) Define hyperplane and give example.

b) Define canonical minimization LPP.

c) Pivot on $a_{12} = 3$ in the following table.

x	y	-1	
2	3	6	$=-t_1$
1	3	2	$=-t_2$
0	2	1	= f

d) State the canonical minimization L.P.P.

x	2	4	6
y	1	0	2
-1	3	2	1
	$=t_1$	$=t_2$	=g

e) Define negative transpose of the minimum table in a L.P.P.

f) Write the matrix reformulation of canonical maximization nd minimization L.P.P.

g) Reduce the following table of the matrix game.

$$\begin{bmatrix} 2 & 1 & 4 & 2 \\ 1 & -2 & 1 & 1 \\ -2 & 6 & 3 & -2 \\ 3 & -3 & 5 & 1 \\ 1 & -2 & 2 & 1 \end{bmatrix}$$

h) Define mixed strategy and pure strategy for a row player of a matrix game.

i) State the Von-Neumann Minimax theorem.

j) Find all the permutation set of zeros of following table of balanced assignment problem:

0	0	1
0	1	0
2	0	0

k) State balanced transportation problem.

l) State VAM.

m) State the maximal flow network problem.

n) Define a path and a cycle in a directed network.

o) Define a cut, cutset and capacity of a cut in a capacitated directed network.

PART - B

UNIT-I

2\. a) An oil company owns two refineries, say refinery A and refinery B. Refinary A is capable of producing 20 barrels of gasoline and 25 barrrels of fuel oil per day: refinery B is capable of producing 40 barrels of gasoline and 20 barrels of fuel oil per day. The company requires at least 1000 barrels of gasoline and at least 800 barrels of fuel oil. If it costs $300 per day to operate refinery A and $500 per day to operate refinery B, how many days should each refinery be operated by the company so as to minimize costs? Solve graphically. **(9)**

b) State the simplex algorithm for maximum basic feasible table. **(9)**

3\. a) Apply simplex algorithm to the maximum table **(9)**

$$\begin{array}{c|cc|c|l}
 & x_1 & x_2 & -1 & \\ \hline
 & -1 & -2 & -3 & = -t_1 \\
 & 1 & 1 & 3 & = -t_2 \\
 & 1 & 1 & 2 & = -t_3 \\ \hline
 & -2 & 4 & 0 & = f \\ \hline
\end{array}$$

b) State the pivot transformation for maximum and minimum table. **(9)**

UNIT-II

4\. a) Solve the non-canonical LPP below:

Maximize $f(x,y) = x + 3y$

subject to $x + 2y \leq 10,$

$3x + y \leq 15$ **(9)**

b) Solve the dual Canonical L.P.P. below. **(9)**

$$\begin{array}{c|cc|c|l}
 & x_1 & x_2 & -1 & \\ \hline
y_1 & 1 & 2 & 20 & = -t_1 \\
y_2 & 2 & 2 & 30 & = -t_2 \\
y_3 & 2 & 1 & 25 & = -t_3 \\ \hline
-1 & 200 & 150 & 0 & = f \\ \hline
 & = s_1 & = s_2 & = g &
\end{array}$$

5\. a) Solve the non-canonical LPP below:

Maximize $f(x,y,z) = x + 2y + z$ **(9)**

subject to $x + y + z = 6$

$x + y \leq 1$

$x, z \geq 0$

b) Prove that a pair of feasible solutions of dual canonical L.P.P. exhibit complementary slackness if and only if they are optimal solutions. **(9)**

UNIT-III

6\. a) Solve the dual non canonical L.P.P. **(9)**

$$\begin{array}{c|cc|c|l}
 & \text{\textcircled{x_1}} & x_2 & -1 & \\ \hline
\text{\textcircled{y_1}} & 2 & -1 & -1 & = -0 \\
y_2 & -1 & 1 & -1 & = -t_1 \\ \hline
-1 & 2 & 1 & 0 & = f \\ \hline
 & = 0 & = s_1 & = g &
\end{array}$$

b) Find the optimal strategies and the Von Neumann value of the matrix game with pay off matrix. **(9)**

$$\begin{bmatrix} 0 & \frac{y}{4} \\ \frac{x-y}{4} & 0 \end{bmatrix}$$

7\. a) Solve the dual non canonical L.P.P. **(9)**

$$\begin{array}{c|cc|c|l}
 & \text{\textcircled{x_1}} & x_2 & -1 & \\ \hline
\text{\textcircled{y_1}} & 1 & -1 & -2 & = -0 \\
y_2 & -2 & 2 & -1 & = -t_1 \\ \hline
-1 & 0 & 1 & 0 & = f \\ \hline
 & = 0 & = s_2 & = g &
\end{array}$$

b) Solve the matrix game $\begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix}$ **(9)**

MAT 501.1 Reg. No. ....

CREDIT BASED FIFTH SEMESTER B.Sc. DEGREE EXAMINATION APRIL 2016

MATHEMATICS

PAPER V – DIFFERENTIAL EQUATIONS AND RING THEORY

Time: 3 Hrs **Max. Marks: 120**

Note: 1. **Answer any TEN questions in Part A. Each question carries 3 marks.**
2. **Answer FIVE full questions from Part B choosing ONE full question from each unit.**

PART A

3x10=30

1. a) Solve $\left(D^2+4\right)y=\cos 2x$.

b) Find a particular integral of the differential equation $\left(D^2-6D+9\right)y=e^x$.

c) Find the complementary function of the differential equation $\left(D^2-8D+9\right)y=8\sin 5x$.

d) Transform $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=x^2$ into a differential equation with constant coefficients using the substitution $z=\log x$.

e) Reduce $y_2-4xy_1+(4x^2-3)y=e^{x^2}$ to normal form.

f) Find A in the method of variation of parameters to solve $\frac{d^2y}{dx^2}+y=\sec x$ if $y=A\cos x+B\sin x$.

g) Find $L\{t\sin kt\}$.

h) For a positive integer n, prove that $L\left\{t^n\right\}=\frac{n!}{s^{n+1}}, s>0$

i) Find $L^{-1}\left\{\frac{s}{s^2+8s+16}\right\}$

j) Let R be a ring. For any a, b in R show that
i) $a\cdot 0=0$ and (ii) $(-a)(-b)=ab$

k) If U is an ideal of a ring R and $1\in u$, then show that U = R.

l) Prove that the ring Z_p of integers modulo p is an integral domain when p is a prime number.

m) Define Euclidean ring.

n) Find all units in *J[i]*

o) Prove that $f(x)=x^2+1$ is irreducible over the integers modulo 7.

PART – B

2. a) Solve $\left(D^2-2D+1\right)y=e^{2x}+e^x$ (6)

b) Solve $\left(D^2+5D+6\right)y=\sin 4x+e^{-2x}$ (6)

c) Solve $\left(D^3+D^2+D+1\right)y=2x^3+3x^2-4x+5$ (6)

3. a) Solve $\left(D^2-4\right)y=\sin^2 x$ (6)

b) Solve $\left(D^2-8D+9\right)y=8\sin 5x$ (6)

c) Solve $\left(D^3-D^2-D+1\right)y=1+x^2$ (6)

UNIT II

4. a) Solve $\left(D^2-2D+4\right)y=e^x\sin x$ (6)

b) Solve $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}-3y=x^2$ (6)

c) Solve $\dfrac{d^2y}{dx^2}+\tan x\dfrac{dy}{dx}+y\cos^2 x=0$ by the method of changing the independent variable. (6)

5. a) Solve $\left(D^2-7D-6\right)y=e^{2x}(1+x)$ (6)

b) Solve $(x+a)^2\dfrac{d^2y}{dx^2}-4(x+a)\dfrac{dy}{dx}+6y=x$ (6)

c) Solve $xy_2+2y_1+xy=0$ by reducing to normal form. (6)

UNIT III

6. a) Derive the formula $L\{F(t)\}=\dfrac{1}{1-e^{-sw}}\int_0^w e^{-s\beta}f(\beta)d\beta$ for the Laplace transform of a periodic function with period w. (6)

b) Find $L^{-1}\left\{\dfrac{s+1}{s^2+6s+25}\right\}$ (6)

c) Solve $y''(t)-y(t)=5\sin 2t,\ y(0)=0,\ y'(0)=1$ (6)

7. a) Derive $L\{\cos kt\}$ (6)

b) Find $F(t)=L^{-1}\left\{\dfrac{e^{-3s}}{(s+1)^3}\right\}$ and hence find $F(2), F(5)$ (6)

c) Express in terms of α-function and find $L\{F(t)\}$ (6)

where $F(t)=\begin{cases}t^2, & 0<t<1\\ 2, & t>1\end{cases}$ (6)

UNIT IV

8. a) Prove that every finite integral domain is a field. (6)

b) If U and V are ideals of a ring R then prove that $U+V=\{u+v/u\ \in u, v\in V\}$ is also an ideal of R. (6)

c) Define $\phi : J\left(\sqrt{2}\right)$ by the rule $\phi\left(m+n\sqrt{2}\right)=m-n\sqrt{2}$. Prove that ϕ is an isomorphism of $J\left(\sqrt{2}\right)$ onto $J\left(\sqrt{2}\right)$. (6)

9. a) Prove that the homomorphism ϕ of R into R^1 is an isomorphism if and only if its kernel, $K_\phi=\{0\}$. (6)

b) If R is a commutative ring R with unit element whose only ideals are (0) and R itself, prove that R is field. (6)

c) If U is an ideal of ring R, prove that $r(U)=\{x\in R / xu=o, \forall\ u\in U\}$ is also an ideal of R. (6)

UNIT V

10. a) If p is a prime number of the form $4n+1$, then prove that $p=a^2+b^2$ for some integers a, b. (6)

b) In the ring Z of integers, prove that $U=(n_0)$ is a maximal ideal If n_0 is a prime number. (6)

c) If $f(x)$ and $g(x)$ are two non-zero elements of $F[x]$, then prove that $\deg\left(f(x)g(x)\right)=\deg f(x)+\deg g(x)$ (6)

11. a) Prove that the element a in a Euclidean ring is a unit if and only if $d(a)=d(1)$ (6)

b) Let R be a commutative ring with unit element and P be an ideal of R. Prove that P is a prime ideal of R if and only if $\dfrac{R}{P}$ is an integral domain. (6)

c) Let R be a Euclidean ring. Prove that any two elements a and b in R have a greatest common divisor 'd' which can be expressed as $d=\lambda a+\mu b$ for some λ, μ in R. (6)

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UNIT-IV

8. a) State the algorithm LARGEST 1 for finding the largest of n numbers. Also justify it with a formal proof. **(9)**

   b) Show that the language $L = \{a^k b^k | k \geq 1\}$ is not a finite state language. **(9)**

Type equation here.

9. a) Let L be a finite state language accepted by a finite state machine with N states. For any sequence α whose length is N or larger in the language. Show that α can be written as uvw ,such that v is nonempty and uv^iw is also in the language for $i \geq 0$, where v^i denotes the concatenation of i copies of the sequence v. **(8)**

   b) State the algorithm 'BUBBLESORT' for sorting the 'n' numbers $x_1, x_2, \ldots\ldots, x_n$. Justify the algorithm with a formal proof. Find its time complexity. **(10)**

10.a) Write the recurrence relation for the Fibonacci sequence of numbers and find its homogeneous solution. **(6)**

   b) Find the particular solution of the difference equation $a_r + a_{r-1} = 3r2^r$ **(6)**

   c) Let a and b numeric function such that $a_r = \begin{cases} 2, & 0 \leq r \leq 3 \\ 2^{-r}+5, & r \geq 4 \end{cases}$ find the forward difference Δa and the backward difference ∇a. **(6)**

11. a) Determine the numeric function corresponding to the generating function

$$A(Z) = \frac{2+3Z-6Z^2}{1-2Z}$$ **(6)**

   b) Find the homogeneous solution of the difference equation:

$$a_r + 6a_{r-1} + 12a_{r-2} + 8a_{r-3} = 0$$ **(6)**

   c) Find the particular solution of the difference equation **(6)**

$$a_r - 5a_{r-1} + 6a_{r-2} = 3r^2$$

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MAT 502.1 Reg. No. ....

CREDIT BASED FIFTH SEMESTER B.Sc. DEGREE EXAMINATION APRIL 2016
MATHEMATICS
PAPER VI – DISCRETE MATHEMATICS

Time: 3 Hrs **Max. Marks: 120**

Note: **1. Answer any TEN questions in Part A. Each question carries 3 marks.**
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A 3x10=30

1. a) If $A = \{1, 2, 3\}$ write P(A). Also find |P(P(A)|

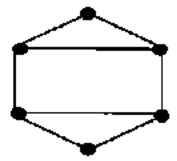
   b) In how many ways can three examinations be scheduled within a five day period so that:
   (i) No two examinations are held on the same day?
   (ii) With no restriction on the number of examinations scheduled for each day.

   c) Prove that $2^n > n^3$ for $n \geq 10$.

   d) Define the terms: (i) Eulerian Circuit (ii) 'Region' in a planar graph.

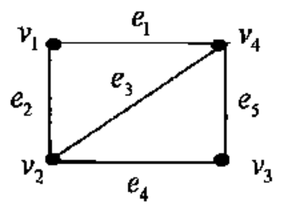
   e) Using Euler's formula show that a complete undirected graph on 5 vertices K_5 is non-planar.

   f) Define k factor. Draw the k-factor of the following graph for k = 1 and k = 2.



   g) Prove that a connected graph always contains a spanning tree.

   h) List all the fundamental cutsets w.r.t. the spanning tree $\{e_1, e_3, e_4\}$ for the following graph.



   i) Obtain a binary tree for the prefix code {1, 01, 000, 001}

   j) Analyze the time complexity of the algorithm LARGEST2.

   k) Draw the state diagram for the modulo 3 sum of the digits {0, 1, 2} in the input sequence.

   l) Prove that two states of a finite state machine are in the same block in π_k if and only if they are in the same block in π_{k-1} and for any input letter, their successors are in the same block in π_{k-1}.

m) Find the backward difference function ∇a for the numeric function.

$$a_r = \begin{cases} 0, & 0 \le r < 2 \\ 2^{-r} + 5, & r \ge 3 \end{cases}$$

n) Define 'generating function' and find the generating function corresponding to the numeric function a for which $a_r = 3^r$.

o) Find $a_r^{(p)}$ for the difference equation $a_r - 2a_{r-1} + a_{r-2} = 7$.

PART B

UNIT I

2. a) Let $< P, \le >$ be a partially ordered set. Suppose the length of the longest chain in P is n. Show that the elements in P can be partitioned into n disjoint antichains. (6)

b) Show by induction that any integer composed of 3^n identical digits is divisible by 3^n. (6)

c) If no three diagonals of a convex decagon meet at the same point inside the decagon, into how many line segments are the diagonals divided by their intersections? (6)

3. a) Define a countably infinite set. Also prove that the set of all real numbers between 0 and 1 is uncountably infinite. (6)

b) Provide a step-by-step derivation to the sentence $C = A + D * (D + B)$ using the following set of productions. (6)

Asgn_Stat → id = exp
exp→ exp + term
exp → term
term → term * factor
term → factor
factor → (exp)
factor → id
id → A
id → B
id → C
id → D

c) If A, B, C be arbitrary sets, then show that $(A - B) - C = (A - C) - B$ (6)

UNIT II

4. a) Let G be a linear graph with n vertices. If the sum of degrees of each pair of vertices in G is n – 1 or larger, then prove that there exists a Hamiltonian path in G. (9)

b) Prove that there is always a Hamiltonian path in a directed complete graph. (9)

5. a) In a directed graph with n vertices, if there is a path from the vertex v_1 to the vertex v_2, then prove that there is a path of not more than (n – 1) edges from vertex v_1 to the vertex v_2. (6)

b) For any connected planar graph prove with usual notations that v – e + r = 2. (6)

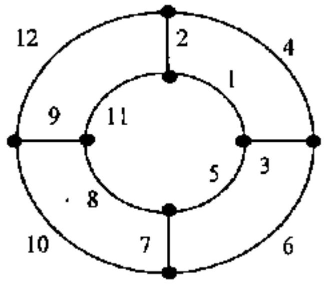
c) Show that a planar graph on n vertices can have atmost 3n – 6 edges. (6)

UNIT III

6. a) Prove that a graph with e = v – 1 edges that has no circuits is a tree. (6)

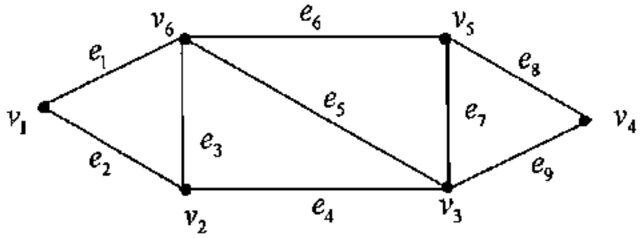
b) For a given spanning tree, let $D = \{e_1, e_2, ..., e_k\}$ be a fundamental cutset in which e_1 is a branch and $e_2, e_3, ..., e_k$ are chords of the spanning tree. Then prove that e_1 is contained in the fundamental circuits corresponding to $e_i, i = 2, 3, ..., k$. Also prove that e_1 is not contained in any other fundamental circuits. (6)

c) Write an algorithm to determine a minimum spanning tree of a connected weighted graph and also obtain minimum spanning tree for the weighted graph given below: (6



7. a) Prove that every circuit has an even number of edges in common with every cut set. (6)

b) Define 'Fundamental circuit' and list all the fundamental circuits with respect to the spanning tree $\{e_1, e_2, e_6, e_4, e_9\}$ (6)



c) Prove that the number of vertices is one more than the number of edges in a tree. (6)