MAT 301.1 **Reg. No. ....**

CREDIT BASED THIRD SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2012

MATHEMATICS

PAPER III: NUMBER THEORY AND DIFFERENTIAL EQUATIONS

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A **3x10=30**

1. a) If þ is a prime and þ|ab then prove that þ|a or þ|b.

b) Find the remainder obtained upon dividing the sum
1! + 2! + 3! + 4! + ------- + 99! + 100! By 12.

c) If a is a solution of $þ(x) \equiv 0 (mod\ n)$ and $a \equiv b\ (mod\ n)$ then show that b is also a solution.

d) Find $\phi\,(26{,}000)$.

e) For n>2, prove that $\phi\,(n)$ is an even integer.

f) If g.c.d. (a, bc) = 1, prove that g.c.d. (a, b) = 1 where a, b, c are integers.

g) If x, y, z is a primitive Pathagorean triple, prove that one of integers x and y is even, while the other is odd.

h) For the Bibonacci sequence $\{\mho_n\}$, show that gcd $(\mho_n, \mho_{n+1}) = 1$ for every $n \geq 1$.

i) Define 4^{th} convergent of a continued fraction and find c_2 of [0; 2, 1, 2, 6]

j) Find the orthogonal trajectories of the family of straight lines with slope and y – interrupt equal.

k) Determine whether the function $f(x, y) = n\, Sin\, \frac{y}{x} - y\, Sin\, \frac{x}{y}$ is homogenous or not. If it is homogenous then find its degree.

l) Test the exactness of $(cos\, x\, cos\, y - \cot x)dx - \sin x \sin y\, dy = 0$

m) Find the þ -discriminant equation of $xp^2 - 2yp + 4x = 0$

n) Solve : $y = px + p^3$

o) Solve : $y^{11} = x(y^1)^3$

PART - B

UNIT-I

2\. a) For arbitrary integers a and b, prove that $a \equiv b\ (mod\ n)$ if an only if a and b leave the same non-negative remainder when divided by n. **(6)**

b) State and prove fundamental theorem of arithmetic. **(6)**

c) Let $N = a_m 10^m + 1_{m-1}10^{m-1} + \cdots + a, 10 + a_0$ be the decimal representation of the positive integer N, $0 \le a_k < 10$ and let s=$a_0 + a_1 + \cdots + a_m$. Prove that 9/N if and only if 9/S. **(6)**

3\. a) Solve the linear congruence: 6 ≡ 15 (mod 21) **(6)**

b) If $ca \equiv cb\ \ (mod\ n)$ then prove that $a \equiv b\ (mod\ {}^{n}/_{d})$ where d = g.c.d. (c, n) **(6)**

c) Let $p(x) = \sum_{k=0}^{m} C_k x^k$ be a polynomial function x with integral coefficients

C_K. If $a \equiv b\ (mod\ n)$ then prove that $p(a) \equiv p(b)(mod\ n)$. **(6)**

UNIT-II

4\. a) If n is a positive integer and g.c.d. (a, n) = 1 then prove that $a^{\phi(n)} \equiv 1\ (mod\ n)$ **(6)**

b) If þ is a prime and þ +a then prove that $a^{p-1} \equiv 1\ (mod$ þ$)$ **(6)**

c) If þ and q are distinct primes such that $a^p \equiv a\ (mod\ q)$ and $a^q \equiv a(mod\ p)$ then prove that $a^{pq} \equiv a\ (mod\ pq)$. **(6)**

5\. a) For $n > 1$, show that the sum of the positive integer less than n and relatively prime to n is $\frac{1}{2}n\phi(n)$. **(6)**

b) Show that the quadratic congruence $x^2 + 1 \equiv 0 \ (mod\, p)$ where p is an odd prime, has a solution if and only if $p \equiv 1 \ (mod\ 4)$. **(6)**

c) If the integer n > 1 has the prime factorization $n = p_1^{k_1}\, p_2^{k_2} \ldots p_n^{k_n}$ then prove that $(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \ldots \left(1 - \frac{1}{p_n}\right)$. **(6)**

UNIT-III

6. a) If $ab = c_n$ where g.c.d. $(a, b) = 1$, then prove that there exist positive integers a_1, b_1 such that $a = a_1^n,\ b = b_1^n$ **(6)**

b) Prove that the g.c.d. of two Fibonacci numbers is again a Fibonacci number. **(6)**

c) Express $\frac{187}{57}$ as finite simple continued fraction. **(6)**

7. a) Prove that the radius of the inscribed circle of a pythagorean triangle is always an integer. **(6)**

b) If $C_k = \frac{p_k}{q_k}$ is the k^{th} convergent of the simple continued fraction $[a_0, a_1, a_2, \ldots\ a_n]$ then prove that $p_k q_{k-1} - q_k p_{k-1 = (-1)^{k-1}}, 1\ \leq k \leq n$. **(6)**

d) Prove that any rational number can be written as a finite simple continued fraction. **(6)**

UNIT-IV

8. a) Solve : $(x^2 + 2xy = 4y^2 dx - (x^2 - 8xy - 4y^2)dy = 0$ **06**

b) Solve : $y\ dx + (3x - xy + 2)dy = 0$ **(6)**

c) Solve : $(2x + 3y - 5)dx + (3x - y - 2)dy = 0$ **(6)**

9. a) Solve $6y^2 dx - x\ (2x^3 + y)\ dy = 0$ **(6)**

b) Solve : $3x^2 y\ dx + (y^4 - x^3)dy = 0$ **(6)**

c) Find the orthogonal trajectories of $r = a(1 + cos\theta)$ **(6)**

UNIT-V

10. a) Solve : $p^2 - (x^2y - 3)p + 3x^2y = 0$

b) Solve : $(x^2 - 1)p^2 - 2xyp + \ y^2 - 1 = 0$

c) Solve : $xy^{11} - (y^1)^3 - y^1 = 0$

11. a) Solve : $xyp^2 + (x + y)p + 1 = 0$

b) Solve : $xp^2 - 3yp + \ 9x^2 = 0,$ for $x > 0$

c) Solve : $yy^{11} - (y^1)^2 + 1 = 0$

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MAT 301.1 **Reg. No. ....**

CREDIT BASED THIRD SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2013

MATHEMATICS

PAPER III: NUMBER THEORY AND DIFFERENTIAL EQUATIONS

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A **3x10=30**

1. a) Prove that 41 divides $2^{20} - 1$

b) If p is a prime and $p|$ab then prove that $p|$a or $p|$b.

p) Solve the linear congruence $6x \equiv 15 \ mod \ 21$

q) If p is a prime, prove that $a^p \equiv a \ mod \ p$ for any integer a.

r) Find the sum of all positive integers which are less than 30 and relatively prime to 30.

s) Find ϕ (1001).

t) If u_n is a Fibonacci sequence find the g.c.d. (u_{16}, u_{12}).

u) Find the convergent C_2 for the simple continued fraction [0; 2, 1, 2, 5, 1].

v) Prove that g.c.d. $(u_n, u_{n+1}) = 1$ for every $n \geq 1$. where u_n is the n[th] Fibonacci number.

w) Solve $x \ Sin \ y \ dx + \ x^2 tan \ y \ dy = 0$

x) Find whether the function $f(x, y) = x \ log \ x - x \ log \ y$ is homogeneous.

y) Find the integrating factor of the differential equation $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$.

z) Solve the equation $p^2 - xp + y = 0$

aa) Solve $(y + 1) \ y^{//} \ = (y^{/})^2$

bb) Find the orthogonal trajectories of the family $y^2 = 4ax$

PART - B

UNIT-I

2. a) State and prove fundamental theorem of Arithmetic. **(9)**

b) If $P(x) = \sum_{k=0}^{m} c_k x^k$ is a polynomial with integral coefficients c_k and $a \equiv b \ mod \ n$, then prove that $P(a) \equiv P(b) \ mod \ n$. **(4)**

c) If $N = \sum_{k=0}^{m} a_k \ 10^k$ is the decimal representation of a positive integer N, $0 \leq a_k < 10$ and T = $a_0 - a_1 + a_2 - a_3 + \cdots + (-1)^m a_m$, then prove that 11/N if and only if 11/T. **(5)**

3. a) Show that the linear congruence $ax \equiv b \ mod \ n$ has a solution iff d | b where d = g.c.d.(a, n). Also prove that if d | b then it has 'd' mutually incongruent solutions modulo n. **(9)**

b) Solve the linear congruence $17x \equiv 9 \ mod \ 276$. **(9)**

UNIT-II

4. a) If p is a prime prove that $(p-1)! \equiv -1 \, mod \, p$ **(9)**

b) If n is a positive integer and g.c.d.(a, n) = 1 then prove that $a^{\phi(n)} \equiv 1 \ (mod \ n)$ **(9)**

5. a) If $n = p_1^{k_1} \, p_2^{k_2} \, \dots \, p_r^{k_r}$ then prove that

$\phi\,(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \dots \left(1 - \frac{1}{p_r}\right)$ where $n > 1$. **(6)**

e) For *n* > *2*, prove that $\phi\,(n)$ is an even integer. **(6)**

f) For *n* > *1* prove that the sum of positive integers less than *n* and relatively prime to *n* is $\frac{1}{2} n\phi(n)$. **(6)**

UNIT-III

6. a) Prove that radius of the inscribed circle of a Pythagorean triangle is always an integer. **(6)**

b) Prove that any rational number can be written as a finite simple continued fraction. **(6)**

d) Prove that for Fibonacci numbers, g.c.d. $(u_m, u_n) = u_d$ where d =g.c.d.(m, n) **(6)**

7. a) Prove that area of a Pythagorean triangle can never be equal to a perfect square. **(6)**

b) Prove that the kth convergent of a simple continued fraction $[a_0; a_1, a_2, \dots \, a_n]$ has the value $C_k = \frac{p_k}{q_k} \ \ 0 \le k \le n$, where $k \ge 2$ & $p_k = a_k p_{k-1} + p_{k-2}$,
$q_k = a_k q_{k-1} + q_{k-2}$ **(6)**

c) Express $\frac{187}{57}$ as a simple continued fraction. **(6)**

UNIT-IV

8. a) Solve $xy \, dx - (x^2 + 3y^2) \, dy = 0$ **(6)**

b) Solve $(2x^3 - xy^2 - 2y + 3) \, dx - \ (x^2 y + 2x) dy = 0$ **(6)**

c) Find the orthogonal trajectories of the family of curves *r = a (1 + cos θ)* **(6)**

9. a) Solve $y(6y^2 - x - 1)dx + 2x \, dy = 0$ **(6)**

b) Solve $y \, dx + (3x - xy + 2)dy = 0$ **(6)**

c) A substance is being converted into another substance. At the end of half a minute, two thirds of the original amount has been already converted. Find how much unconverted substance remains at t = 60 seconds. **(6)**

UNIT-V

10. a) Solve $xyp^2 + (x + y)p + 1 = 0$ **(6)**

b) Find the general solution and also the singular solution of the equation $xp^2 - 2yp + 4x = 0$ **(6)**

c) Solve : $2yy^{//} + (y^{/})^3 = 0$ **(6)**

11. a) Find the general and singular solution of $p^2 + x^3p - 2x^2y = 0$ **(6)**

b) Solve $y^{//} = x(y^{/})^3$ **(6)**

c) Solve $p^2 - (x^2y + 3)p + 3x^2y = 0$ **(6)**

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MAT 301.1 **Reg. No. ....**

CREDIT BASED THIRD SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2014

MATHEMATICS

PAPER III: NUMBER THEORY AND DIFFERENTIAL EQUATIONS

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.

2. **Answer FIVE full questions from Part B choosing ONE full question from each unit.**

PART A **3x10=30**

1. a) Find the remainder when the sum $1!+2!+3!+...+99!+100!$ is divided by 12.

b) Using divisibility test, find whether the number 457182 is divisible by 11 or not.

cc) Solve the congruence $5x \equiv 2 \ (\text{mod } 26)$.

dd) If p is a prime, then prove that $a^p \equiv a \ (\text{mod } p)$, for any integer a.

ee) For $n > 2$, prove that $\phi(n)$ is an even integer.

ff) Calculate ϕ (5040).

gg) *If x, y, z* is a primitive Pythagorean triple. Then prove that one of integers x and y is even while the other is odd.

hh) For the Fibonacci sequence, prove that $g.c.d.(u_n, u_{n+1}) = 1$ for every $n \geq 1$.

ii) Express $[3, 2, 1, 2, 5, 1]$ as a rational number.

jj) Determine whether the function $f(x, y) = x \sin {}^{y}\!/\!_{x} - y \sin {}^{y}\!/\!_{x}$

kk) Check the exactness of $y^2 - 2xy + 6x \ \ dx - \ \ x^2 - 2xy + 2 \ \ dy = 0$

ll) Find the integrating factor of the differential equation
$y(x+y)dx + (x+y-1)dy = 0$

mm) Find the orthogonal trajectories of $x - 4y = c$

nn) Solve $x^2p^2 - y^2 = 0$

oo) Solve $y = px + p^3$

PART - B

UNIT-I

2. a) Prove that every positive integer $n > 1$ can be expressed as a product of primes, this representation is unique apart from the order in which factors occur. **(6)**

b) If $N = \sum_{k=0}^{m} a_n 10^k$ is the decimal representation of a positive integer $N, 0 \leq a_k < 10$ and $S = a_0 + a_1 + ... + a_m$, then show that $9 | N$ if and only if $9 | S$.

$a \equiv b \ mod \ n$, then prove that $P(a) \equiv P(b) \ mod \ n$. **(6)**

c) Solve the following simultaneous congruences
$x \equiv 1 \ (\text{mod} 3), \ x \equiv 2 \ (\text{mod} 5), \ x \equiv 3 \ (\text{mod} 7)$ **(6)**

3. a) Show that the linear congruence $ax \equiv b \bmod n$ has a solution iff d | b where d = g.c.d.(a, n). Also prove that if d | b then it has 'd' mutually incongruent solutions modulo n. **(6)**

b) If $P(x) = \sum_{k=0}^{n} C_k x^k$ is a polynomial with integral coefficients C_k and $a \equiv b \pmod n$ then prove that $P(a) \equiv P(b) \bmod n$. **(6)**

c) If p is a prime and $p \mid ab$ then prove that $p \mid a$ or $p \mid b$. **(6)**

UNIT-II

4. a) If n is a positive integer and g.c.d.(a, n) = 1 then prove that $a^{\phi(n)} \equiv 1 \ (mod\ n)$, where $\phi(n)$ is Euler's function ϕ. **(9)**

b) Prove that the quadralic congruence $x^2 + 1 \equiv 0 \pmod p$ **(9)**

5. a) If p is a prime and $p \chi a$ then prove that $a^{p-1} \equiv 1 \pmod p$ **(6)**

g) Given integers a, b, c prove that $g.c.d.(a, bc) = 1$ if and only if $g.c.d.(a, b) = 1$ and $g.c.d.(a, c) = 1$. **(6)**

h) For each positive $n \geq 1$, show that $n = \sum_{d|n} \phi(d)$, the sum being extended over all positive divisors of *n*. **(6)**

UNIT-III

6. a) If $m = qn + r$ then prove that $g.c.d\ u_m, u_n = g.c.d\ u_n, u_r$ **(6)**

b) Prove that any rational number can be written as a finite simple continued fraction. **(6)**

e) Express $\frac{187}{57}$ as a finite simple continued fraction. **(6)**

7. a) Prove that radius of the inscribed circle of a Pythagorean triangle is always an integer. **(6)**

b) For $m \geq 1, n \geq 1$, show that U_{mn} is divisible by U_m **(6)**

c) If $C_k = \frac{p_k}{q_k}$, is the k^{th} convergent of the simple continued fraction $[a_0, a_1, a_2, ... a_n]$ then prove that $p_k q_{k-1} - q_k p_{k-1} = (-1)^{k-1}, 1 \le k \le c$ **(6)**

UNIT-IV

8. a) Solve $3\ \ 3x^2 + y^2\ \ dx - 2xydy = 0$ **(6)**

b) Solve $\frac{dy}{dx} = y - xy^3 e^{-2x}$ **(6)**

c) Find the orthogonal trajectories of the family of curves given by $r = a(1 + \sin\theta)$ **(6)**

9. a) Solve $y^1 = \cos ecx + y\cot x$. **(6)**

b) Solve $y = \ \ 6y^2 - x - 1)dx + 2xdy = 0$ **(6)**

c) A certain radio-active substances has a half-life of 38 hours. Find how long it takes for 90% of the radioactivity to be dissipated? **(6)**

UNIT-V

10. a) Solve $x^2 p^2 - 5xyp + 6y^2 = 0$ **(6)**

b) Find the general and singular solution of $p^2 + x^3 p - 2x^2 y = 0$ **(6)**

c) Solve $\ x^2 - 1\ \ p^2 - 2xyp + y^2 - 1 = 0$ **(6)**

11. a) Solve $xp^2 - (2x \_ 3y)p + 6y = 0$ **(6)**

b) Solve $xp^2 - 3yp + 9x^2 = 0$ **(6)**

c) Solve $xy^{11} - \ge\ \ y^{1\ \ 3} - y^1 = 0$ **(6)**

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MAT 301.2 **Reg. No. ....**

CREDIT BASED THIRD SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2015

MATHEMATICS

PAPER III: FUNCTIONS OF SEVERAL VARIABLES,MULTIPLE INTEGRALS AND GROUP THEORY

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A **3x10=30**

1. a) Find the domain of $z = \dfrac{1}{\sqrt{x^2 + y^2 - 25}}$

b) Find the slope of the tangent line to the curve of intersection of the surface $z = \dfrac{1}{2}\sqrt{24 - x^2 - 2y^2}$ with the plane y = 2 at the point 2, 2, $\sqrt{3}$

pp) If $f(x, y) = e^x \sin y + \ln xy$, find $\dfrac{\partial^3 f}{\partial x\, \partial y^2}$

qq) Find by double integration the area of the region in the xy plane bounded by the curves $y = x^2$ and $y = 4x - x^2$

rr) Evaluate $\iint_R e^{-(x^2+y^2)} dA$, where the region R is in the first quadrant and bounded by the circle $x^2 + y^2 = a^2$ and the coordinate axes.

ss) Find the area of the surface that is cut from the cylinder $x^2 + z^2 = 16$ by the planes $x = 0, x = 2, y = 0$ and $y = 3$.

tt) Evaluate $\int_0^1 \int_0^{1-x} \int_{2y}^{1+y^2} xdz\, dy\, dx$.

uu) Evaluate the line integral $\int_C 3xdx + 2xydy + zdz$
where $C: x = \cos t,\ y = \sin t,\ z = t,\ 0 \le t \le 2\pi$

vv) Evaluate the iterated integral $\int_0^{\pi} \int_2^4 \int_0^1 re^z dz\, dr\, d\theta$

ww) Show that every group of prime order is cyclic.

xx) Compute $a^{-1}ba$ where $a = (1\,3\,5)(1\,2)$ and $b = (1\,5\,7\,9)$

yy) Prove that the intersection of two subgroups of a group G is a subgroup of G.

zz) If N is a normal subgroup of G, then show that $gNg^{-1} = N$ for every $g\ \varepsilon\ G$

aaa) Let G be the group of integers under addition. Prove that $\phi : G \to G$ defined by $\phi(x) = 2x$ is a homomorphism and find its Kernel.

bbb) Define i) Centre of group ii) Automorphism

PART - B

UNIT-I

2. a) Prove that $\lim_{(x,y)\to(1,2)} 3x^2 + y = 5$ by applying $\varepsilon - \delta$ definition. **(6)**

b) If $u = x^2 + xy, \quad x = r^2 + s^2, \quad y = 3r - 2s$, find $\frac{\partial u}{\partial r}$ and $\frac{\partial u}{\partial s}$ using chain rule. **(6)**

c) Find the equation of the tangent plane and the equation of the normal line to the surface $x^2 + y^2 + z^2 = 17$ at the point $(2, -2, 3)$ **(6)**

3. a) Let the function f be defined by

$$f(x, y) = \begin{cases} x^2 + y^2, & if\ x^2 + y^2 \le 1 \\ 0 \quad , & if\ x^2 + y^2 > 1 \end{cases}$$

Discuss the continuity of f. What is the region of continuity of f? **(6)**

b) If $f(x, y, z) = y^2 + z^2 - 4xz,$ find the rate of change of $f(x, y, z)$ at $(-2, 1, 3)$ in the direction of the vector $\frac{2}{7}i - \frac{6}{7}j + \frac{3}{7}k$ **(6)**

c) If $f(x, y) = x^3 + y^2 - 6x^2 + y - 1$, then determine the relative extrema of f if there are any. **(6)**

UNIT-II

4. a) Find an approximate value of the double integral $\iint_R x^2 + y \; dA$, where R is the rectangular region having vertices P(0, 0) and Q(4, 2). Take the partition of R formed by the lines x=1, x=2, x=3 and y=1. **(6)**

b) Using double integration find the area of the region inside the cardioid $r = 2(1 + \sin\theta)$. **(6)**

c) Find the area of the top half of the sphere $x^2 + y^2 + z^2 = a^2$ using double integration. **(6)**

5. a) Evaluate $\iint_R x^2\sqrt{9 - y^2}\; dA$ where R is the region bounded by the circle $x^2 + y^2 = 9$ **(6)**

i) Find the volume of the solid in the first octant bounded by the cone z = r and the cylinder $r = 3\sin\theta$ **(6)**

j) Find the area of the paraboloid $z = x^2 + y^2$ below the plane $z = 4$. **(6)**

UNIT-III

6. a) Find the volume of the solid above the elliptic paraboloid $3x^2 + y^2 = z$ and below the cylinder $x^2 + z = 4$ **(6)**

b) Evaluate the line integral $\int_C \overrightarrow{F} \bullet \overrightarrow{dR}$ where

$F(x, y) = 2xy\hat{i} + (x - 2y)\hat{j}, C: R(t) = \sin t\,\hat{i} - 2\cos t\,\hat{j},\ 0 \le t \le \pi$ **(6)**

f) A homogeneous solid in the shape of a right circular cylinder has a radius of 2m and an altitude of 4m. Find the moment of intertia of the solid with respect to its axis. **(6)**

7. a) Evaluate $\int_0^{\pi/4} \int_0^a \int_0^{r\cos\theta} r\sec^3\theta\, dz\, dr\, d\theta$ **(6)**

b) Find the volume of the solid enclosed by the sphere $x^2 + y^2 + z^2 = a^2$ by using spherical coordinates. **(6)**

c) Evaluate the line integral $\int_C x^2 + xy\ \ dx + \ y^2 - xy\ \ dy$

C : the line $y = x$ from the origin to the point (2, 2). **(6)**

UNIT-IV

8. a) If H is a nonempty finite subset of a group G and H is closed under multiplication, prove that H is a subgroup of G. **(6)**

b) State and prove Lagrange's theorem. **(6)**

c) If H and K are finite subgroups of G of orders O(H) and O(K) respectively, prove that $O(HK) = \dfrac{O(H)O(K)}{O(H \cap K)}$

9. a) Let G be a group and H be a subgroup of G. For all $a\,\varepsilon\, G$, prove that $Ha = \{x\,\varepsilon\, G\,|\,a \equiv x(\bmod H)\}$ **(6)**

b) Show that every permutation is the product of its disjoint cycles. **(6)**

c) If G is a group in which $(a \cdot b)^i = a^i \cdot b^i$ for three consecutive integers i for all $a, b\,\varepsilon\, G$, show that G is abelian **(6)**

UNIT-V

10. a) Prove that a subgroup N of a group G is a normal subgroup of G if and only if the product of any two right cosets of N in G is again a right coset of N in G. **(6)**

b) If G and $\overline{G}$ are groups and $\phi: G \to \overline{G}$ is a homomorphism. Prove that

(i) $\phi(e) = \overline{e}$, where e and $\overline{e}$ are identities in G and $\overline{G}$ respectively.

(ii) $\phi(x^{-1}) = \ \phi(x)\ ^{-1} \forall\, x\,\varepsilon\, G$ **(6)**

c) Prove that the set of all automorphisms of a group G is a group. **(6)**

11. a) If G is a group and N is a normal subgroup of G, then prove that G/N is also a group. **(6)**

b) Prove that a homomorphism of G onto $\overline{G}$ with kernel K is an isomorphism of G onto $\overline{G}$ if and only if $K = \{e\}$. **(6)**

c) Prove that S_n has a normal subgroup of index 2, the alternating group A_n consisting of all even permutations. **(6)**

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o) Let G be the group of all positive real numbers under multiplication and G' be the group of all real numbers under addition. Define $\phi : G \to G'$ by $\phi(x) = \log_{10} x$. Show that ϕ is a homomorphism.

PART - B

UNIT-I

2. a) By using $\varepsilon - \delta$ definition, prove that $\lim\limits_{(x,y)\to(1,3)} 2x + 3y = 11$ (6)

b) The temperature at any point (x, y) of a rectangular plate lying in the xy plane is determined by $T(x, y) = x^2 + y^2$.

(i) Find the rate of change of the temperature at the point (3, 4) in the direction making an angle of radian measure $\frac{1}{3}\pi$ with the positive x direction.

(ii) Find the direction for which the rate of change of the temperature at the point $(-3, 1)$ is a maximum. (6)

c) Find an equation of the tangent line to the curve of intersection of $x^2 + y^2 - z = 8, \quad x - y^2 + z^2 = -2$ at the point (2, −2, 0). (6)

3. a) Let f be a function defined by $f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2} & \text{if } (x\ y) \neq (0, 0) \\ 0 & \text{if } (x\ y) = (0, 0) \end{cases}$

find $f_{12}\,(0, 0)$ (6)

b) Given $u = xy + xz + yz, \quad x = r, \quad y = r\cos t, \quad z = r \sin t$, find $\frac{\partial u}{\partial r}$ and $\frac{\partial u}{\partial t}$ using chain rule. (6)

c) If $f(x, y) = 2x^4 + y^2 - x^2 - 2y$ determine the relative extrema of f if there are any. (6)

UNIT-II

4. a) Find an approximate value of the double integral $\iint\limits_R \left(2x^2 - 3y\right) dA$, where R is the rectangular region having vertices $(-1, 1)$ and (2, 3). Take a partition of R formed by the lines $x = 0, \ x = 1$ and $y = 2$ and take $(\xi_i\ \gamma_i)$ at the center of the i^{th} sub region. (6)

b) Find the volume of the solid in the first octant bounded by the two cylinders $x^2 + y^2 = 4$ and $x^2 + z^2 = 4$. (6)

c) Find the area of the paraboloid $z = x^2 + y^2$ below the plane z = 4. (6)

5. a) Find the volume of the solid bounded by the surface $f(x, y) = 4 - \frac{1}{9}x^2 - \frac{1}{16}y^2$, the planes $x = 3$ and $y = 2$ and the coordinate planes. (6)

b) Find by double integration the area of the region inside the cardioid $r = a(1 + \cos\theta)$ and outside the circle $r = a$ (6)

c) Find $\int_1^4 \int_{y^2}^{y} \sqrt{\frac{y}{x}}\, dx\, dy$ (6)

UNIT-III

6. a) Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 25$, the plane $x + y + z = 8$ and xy plane, using triple integrals. (6)

b) Suppose a particle moves along the parabola $y = x^2$ from the point $(-1, 1)$ to the point (2, 4). Find the total work done if the motion is caused by the force field $F(x, y) = \left(x^2 + y^2\right)\hat{i} + 3x^2 y\hat{j}$. Assume that the arc is measured in meters and the force is measured in newtons. (6)

c) A homogeneous solid is bounded above by the sphere $\rho = a$ and below by the cone, $\phi = \alpha, \ 0 < \alpha < \frac{1}{2}\pi$. Find the moment of inertia of the solid about the z – axis. (6)

7. a) A particle transverses the twisted cubic $\vec{R}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}, \ 0 \le t \le 1$. Find the total work done if the motion is caused by the force field $\vec{F}(x, y, z) = e^x\hat{i} + xe^z\hat{j} + x\sin \pi y^2\hat{k}$. Assume that the arc is measured in meters and force is measured in newtons. (6)

b) Evaluate $\int\limits_{-1}^{0}\int\limits_{e}^{2e}\int\limits_{0}^{\pi/3} y \ln z \tan x \ dx\ dz\ dy$ (6)

d) Find the volume of the solid bounded by the paraboloid $x^2 + y^2 + z^2 = 12$ and the plane z = 8 using cylindrical coordinates. (6)

UNIT-IV

8. a) If H is a non empty finite subset of G and H is closed under multiplication then prove that H is a subgroup of G. (6)

b) If G is a finite group and H is a subgroup of G, then show that O(H) | O(G). (6)

c) If H and K are finite subgroups of G of orders O(H) and O(K) respectively, prove that $O(HK) = \dfrac{O(H)O(K)}{O(H \cap K)}$. (6)

9. a) In a group G, define normalizer N(a) of $a \ \varepsilon\ G$ and prove that it is a subgroup of G. (6)

b) If G is a group and H is a subgroup of G, then show that the relation $a \equiv b (\operatorname{mod} H)$ is an equivalence relation. (6)

c) Let H and K be two subgroups of a group G. Prove that HK is a subgroup of G if and only if HK = KH. (6)

P.T.O.

UNIT-V

10. a) Prove that N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \varepsilon G$. (6)

b) If ϕ is a homomorphism of a group G onto $\overline{G}$ with Kernel K, then prove that $\frac{G}{K}$ is isomorphic to $\overline{G}$. (6)

c) Prove that the kernel of a homomorphism is a normal subgroup of group G. (6)

11. a) Define centre of a group. Prove that it is always a normal subgroup. (6)

b) Let G be any group, define $\tau_g : G \to G$ by $\tau_g(x) = g^{-1}xg$. Prove that τ_g is an automorphism. (6)

c) Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G. (6)

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MAT 301.2 Reg. No. ....

CREDIT BASED THIRD SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2016

MATHEMATICS

PAPER III: FUNCTIONS OF SEVERAL VARIABLES, MULTIPLE INTEGRALS AND GROUP THEORY

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A 3x10=30

1. a) Find the domain of $f(x,y) = \frac{\sqrt{25-x^2-y^2}}{x}$

b) If $f(r,\theta) = r\tan\theta - r^2\sin\theta$ find $f_2(3,\pi)$.

c) If $f(x,y) = x^2 - 4y$, find $\nabla f(-2,2)$.

d) Evaluate $\int_0^4\int_0^y \sqrt{9+y^2}\,dx\,dy$

e) Find the volume of the solid in the first octant bounded by the cone z = r and the cylinder $r = 3\sin\theta$

f) Find the area of the surface cut from the plane $2x+y+z=4$ by the planes x = 0, x = 1, y = 0 and y = 1.

g) Evaluate $\iiint_S xy \sin yz\, dV$ if S is the rectangular parallelpiped bounded by the planes $x=\pi,\ y=\frac{1}{2}\pi,\ z=\frac{1}{3}\pi$ and the coordinate planes.

h) Evaluate the line integral $\int_C (x^2+xy)\,dx + (y^2-xy)\,dy$, C : the line y = x from the origin to the point (2, 2)

i) Evaluate the iterated integral $\int_0^{\pi/4}\int_0^{2a\cos\phi}\int_0^{2\pi} \rho^2 \sin\phi\, d\theta\, d\rho\, d\phi$.

j) If G is a finite group and $a \varepsilon G$, then prove that $a^{0(G)} = e$.

k) If H and K are subgroups of G and $O(H) > \sqrt{O(G)}$ and $O(K) > \sqrt{O(G)}$, then prove that $H \cap K \neq (e)$.

l) Find the order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$

m) If G and G' are groups and $\phi : G \to G'$ is an isomorphism, then show that $\ker\phi = \{e\}$, where e is the identity in G.

n) Show that the intersection of two normal subgroups of G is a normal subgroup of G.