MAT 101.1 **Reg. No. ....**

CREDIT BASED FIRST SEMESTER B.Sc. DEGREE EXAMINATION
OCTOBER 2012

MATHEMATICS

PAPER I: CALCULUS AND NUMBER THEORY

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A

3x10=30

1. a) Find the critical numbers of the function $f(x) = x^{2/3} - 2x^{1/3}$

b) Determine the intervals on which $f(x) = 2x^3 - 9x^2 +$ is increasing.

c) Find the absolute maximum and absolute minimum of the function

$f(x) = x^3 + 5x - 4$ on [-3, -1)

d) If $f(x) = (1 - 2x)^3$, find the region in which its graph is concave upward.

e) Find the vertical asymptotes of the graph of the function $f(x) = \frac{4x^2}{x^2}$.

f) Find the points of inflection of $g(x) = x^4 - 8x^2$

g) Find the Cartesian equation of the graph had the polar equation $r^2 = 2\sin 2\theta$

h) Evaluate $\lim_{x \to o^+} \frac{e^{-\frac{1}{2}}}{x}$ if it exists.

i) Find all values of z satisfying the Cauchys mean value theorem for $f(x) = Sinx$ and $g(x) = Cosx$, $(a,b) = (0, \pi$

5 $\int_2^{\frac{\pi}{2}} Cos^7 x\, d$

j) Find

k) Evaluate $\int \sin^3 x \cos^4 x \, dx$

l) Evaluate $\int \frac{x\,dx}{\sqrt{1+x^3}}$

m) Show that square of any odd integer is of the form *8k+1*

n) If a = *bq* + r then prove that *gcd (a, b) = gcd (b, r)*

o) If a|bc with *gcd (ab, b)=1* then prove that a|c.

PART - B

UNIT-I

2. a) If *f (x)* exists for all x in the open interval (a, b) and if *f* has a relative minimum at *c* where *a < c < b* there if $f^1(c)$ exists, then prove that $f^1(c) = 0$ **(6)**

b) A cardboard manufacturer wishes to make open boxes from pieces of cardboard 12 in square by cutting equal squares from the four corners and turning up the sides. Find the length of the side of the square to be cut out to obtain a box of larges possible volume. **(6)**

c) If $f(x) = x^3 - 6x^2 + 9x + 1$ find the relative extremum of *f* by first derivative test. Also determine the intervals on which *f* is increasing and decreasing. **(6)**

3. a) State and prove Rolles Theorem. **(6)**

b) A rectangular field is to be fenced off along bank of a river, no fence is requiredalong the river. If the material for the fence case $ 8 per running foot for the two ends and $ 12 for running foot for the side parallel to the river, find the dimension of the field of largest possible area that can be enclosed with $ 3600 worth of fence. **(6)**

c) If $f(x) = x^3 - x^2 + x$ find the relative extremum of *f* by first derivative test. Also determine the intervals on which *f* is increasing and decreasing. **(6)**

UNIT-II

4. a) Let C be a critical number of a function at which $f^1(c) = 0$ and let $f^{11}(x)$ exists for all values of x in some open interval containing e. Then if $f^{11}(c)$ exists and $f^{11}(c) < 0$, prove that *f* has a relative maximum value at c. **(9)**

b) Find all the asymptotes of the graph of the function $f(x) = \frac{x^2 + [illegible]}{x}$ **(9)**

5. a) If $f(x) = (1 - 2x)^3$ find the point of inflection of the graph of *f(x)* and determine where the graph is concave upwards and concave downloads. **(6)**

b) Sketch the graph of $f(x) = x^3 - 3x^2 + 3$. **(6)**

UNIT-III

6. a) If f and g are functions such that

i) f and g are continuous on the closed interval [a, b]

ii) f and g are differentiable on the open interval (a, b)

iii) for all x in the open interval (a, b), $g^1(x) \neq 0$ they prove that there exist

a number $z \in (a, b)$ such that $\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(z)}{g'(z)}$ **(9)**

b) Evaluate i) $\lim_{x \to \infty} \left(\frac{1}{x^2} - \frac{1}{x^2 \sec x} \right)$

ii) $\lim_{x \to \infty} \frac{\sin\left(\frac{1}{x}\right)}{\tan^{-1}\left(\frac{1}{x}\right)}$ **(3+2)**

c) Find a Taylor polynomial of degree four at x=0 for the function $f(x) = \sin x$ **(4)**

7. a) Let f and g be functions that are differentiable on an open interval I except possibly at the member a and I. Suppose that for all $x \neq a$ in I $g^1(x) \neq 0$

.

Than if $\lim_{x \to \infty} f(x) = 0$ and $\lim_{x \to \infty} g(x) = 0$ and if $\lim_{x \to \infty} \frac{f^1(x)}{g^1(x)} = l$

Prove that $= \lim_{x \to \infty} \frac{f(x)}{g(x)}$

(9)

b) Evaluate $\lim_{x \to \infty} = (2 + 1)^{Cotx}$

(3)

c) Draw a sketch of the four leaved rose $r = 4\ Cos\ 2\theta$

(6)

UNIT-IV

8. a) Obtain a reduction formula for $\int Sin^{n7} x\, dx$ Also evaluate $\int_0^{\pi/2} Sin^n x\, dx$

(8)

b) Evaluate $\int \frac{\sqrt{x}\, dx}{1 + 3\sqrt{x}}$

(6)

c) Evaluate $\int tan^3 x\, Sec^3 x\, dx$

(4)

9. a) Obtain a reduction formula for $\int x^m (\log x)^n dx$

Hence evaluate $\int_0^1 x^4 (\log x)^3 dx$

(7)

b) Evaluate $\int \frac{dx}{x\sqrt{1 + x + x^2}}$

(7)

8 $\int Cos\sqrt{x}\, dx$ $\int Sin\ 3x\ Cos\ 2x\, dx$

b) Evaluate i) ii)
(4)
06

UNIT-V

10. a) Gien integers *a* and *b* with $b > 0$ prove that then exist unique integers *q* and *r*
such that $a = bq + r, \quad 0 \leq r < b$
(8)

b) If *a* and *b* are any positive integers then prove that there exists a positive integer *n*
such that $na \geq b$.
(4)

c) Solve the linear Diophantine equation $172x + 20y = 1000$.
(6)

11. a) Let S be a set of positive integers with the properties

i) $1 \in S$ ii) $k \in S \Rightarrow k + 1 \in S$

Then prove that S is the set of all positive integers.
(6)

b) Given integers a and b not both zero prove that there exists integers x and y
such that $gcd\,(a, b) = ax + by$
(6)

c) If a cock is worth 5 coins, a hen 3 coins and 3 chickens together 1 coin, how
many cocks, hens and chickens totaling 100 can be bought for 100 coins.
(6)

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MAT 101.1 **Reg. No. ....**

CREDIT BASED FIRST SEMESTER B.Sc. DEGREE EXAMINATION
OCTOBER 2013

MATHEMATICS

PAPER I: CALCULUS AND NUMBER THEORY

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A

3x10=30

1. a) Find the critical numbers of the function $g(x) = x^{6/5} - 12x^{1/5}$.

b) Determine the intervals on which $f(x) = x^{1/3} + 4x$ is increasing.

p) Find a suitable value of c such that the conclusion of the mean value theorem is satisfied by the function $f(x) = x^3 - x^2 - x$ on the closed interval $[-2, 1]$.

q) Find the points of inflection of $f(x) = x^3 - 6x^2 + 9x + 1$.

r) If $f(x) = x^{1/3}$ determine where the graph is concave upward.

s) Find the vertical and horizontal asymptotes of the graph of $f(x) = \frac{2x+7}{x-}$.

t) Find the polar equation of a graph having Cartesian equation $x^2 + y^2 - 4x = 0$

u) Find the third degree Taylor polynomial of the function $f(x) = x^{3/2}$ at $x = 4$.

v) Find $\lim_{x \to 0} (\sin x)$.

w) Evaluate $\int \tan^6 x \sec^4 x \, dx$

x) Find $\int \sin^3 x \, dx$

y) Evaluate $\int \frac{x}{3+} dx$

z) If a = bq + r, then prove that g.c.d. (a, b) = g.c.d. (b, r)

aa) Show that the square of any odd integer is of the form 8k + 1.

bb) Show that the square of an integer leaves the remainder 0 or 1 when divided by 4.

PART - B

UNIT-I

2. a) If *f (x)* exists for all x in the open interval (a, b) and if *f(x)* has a relative minimum at *c* where *a* < *c* < *b* then if $f'(c)$ exists, then prove that $f'(c) = 0$ **(6)**

b) State and prove Rolle's theorem. **(6)**

c) Find the relative extrema of $f(x) = x^3 - x^2 - x$ by applying the first-derivative test. Also determine the intervals on which *f* is increasing and the intervals on which f is decreasing. **(6)**

3. a) Let the function f be continuous on the closed interval [a, b] and differentiable on the open interval (a, b). If $f'(x) > 0$ for all *x* in (a, b) then prove that *f* is increasing on [a, b]. **(6)**

b) State and prove Mean value theorem. **(6)**

c) A cardboard box manufacturer wishes to make open boxes from pieces of cardboard 12 inches square by cutting equal squares from the four corners and turning up the sides. Find the length of the side of the square to be cut out to obtain a box of largest possible volume. **(6)**

UNIT-II

4. a) Find the points of inflection of the graph of $f(x) = x^{2/3} - 2x^{1/3}$. Also determine where the graph is concave upward and concave downward. **(6)**

b) If $f(x) = -4x^3 + 3x^2 + 18x$ find the relative extrema of *f(x)* by using the second derivative test. **(6)**

c) Draw a sketch of the graph of the function $f(x) = x^3 - 3x^2 + 3$.
(6)

5. a) Let c be a critical number of a function *f* at which $f'(c) = 0$ and let f' exist for all values *x* in some open interval containing c. If $f''(c)$ exists and if $f''(c) < 0$, then prove that f has a relative maximum at c.
(6)

d) Find all the asymptotes of the graph of the function $f(x) = \frac{x^2 - \cdots}{x}$ **(6)**

e) Sketch the graph of the function $f(x) = \frac{x^2}{x^2}$ **(6)**

UNIT-III

6. a) State and prove Cauchy's mean value theorem
(6)

b) Find the fourth degree Taylor polynomial of $f(x) = \frac{1}{1+x}$ at $x_0 = \cdots$
(6)

c) Draw a sketch of the graph of $r = 2 + 2\cos\theta$
(6)

7. a) Evaluate i) $\lim_{x \to 0}\left(\frac{1}{x^2} - \frac{1}{x^2 \sec \cdots}\right)$

ii) $\lim_{x \to 0}(x+1)^{\cot x}$
(9)

iii) $\lim_{x \to \pi/2} \frac{\log(\sin x)}{(\pi - 2x)^2}$

b) Let *f(x)* and *g(x)* be differentiable in an open interval I, except possibly at $x = a$. Suppose that for $x \neq a$ in I $g'(x) \neq 0$.

Then if $\lim_{x \to a} f(x) = 0 = \lim_{x \to a} g(x)$ and if $\lim_{x \to a} \frac{f'(x)}{g'(x)} = L$

Prove that $\lim_{x \to a} \frac{f(x)}{g(x)} = L$
(9)

UNIT-IV

8. a) When r is any real number and $r \neq -1$, prove that

$$\int x^r \log x\, dx = \frac{x^{r+1}}{r+1}\log x - \frac{x^{r+1}}{(r+1)^2} + C$$

hence evaluate $\int x^3 \log x\, dx$

(6)

b) Find $\int \sin^m x \cos^n x\, dx$

(6)

c) Evaluate $\int x^5 \sqrt{x^2+4}\, dx$
(6)

9. a) Obtain a reduction formula for $\int \tan^n x\, dx$
(6)

b) Evaluate $\int_0^1 x^4 (\log x)^3\, dx$
(6)

c) Evaluate $\int \frac{dx}{x\sqrt{1+x+x^2}}$
(6)

UNIT-V

10. a) Given integers a and b with $b > 0$ prove that there exist unique integers q and r
such that $a = bq + r, \quad 0 \leq r < b$
(6)

b) State and prove Archimedian property.
(6)

c) Let S be a set of positive integers with the properties i) $1 \in S$ ii) whenever the integer k is in S, then the next integer k + 1 must also be in S. Prove that S is the set of all positive integers.
(6)

11. a) Prove that the Diophantine equation ax + by = c has a solution if and only if d | c where d = g.c.d. (a, b). If (x_0, y_0) is any solution, then prove that all other solutions are given by $x = x_0 + \frac{b}{d}t$, $y = y_0 - \frac{a}{d}t$ for varying integer t. **(9)**

b) Find the solution in the positive integers of the Diophantine equation $54x + 21y = 906$. **(9)**

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MAT 101.2 **Reg. No.** ....

CREDIT BASED FIRST SEMESTER B.Sc. DEGREE EXAMINATION
OCTOBER 2014

MATHEMATICS

PAPER I: CALCULUS AND NUMBER THEORY

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A
3x10=30

1. a) Find the dimensions of the largest rectangular garden that can be fenced off with 100 ft of fencing of material.

b) Find the interval on which the curve .

cc) Find the point of inflection of a function

dd) Find a polar equation of

ee) Evaluate —

ff) Derive Maclaurive polynomial of of degree 3.

gg) Evaluate .

hh) Derive the reduction formula for

ii) Evaluate

jj) The region between the curve $\sqrt{\ }$ and the x axis is revolved about x axis to generate a solid. Find it's volume.

kk) Find the length of the curve – from to

ll) Find the area of the region bounded by the group of

mm) Find whether has a solution or not.

nn) If and and prove that .

oo) If and prove that

PART - B

UNIT-I

2. a) State and prove Rolle's theorem. **(6)**

   b) If the function has a local maximum or minimum at a point C in its domain and if is defined at C, prove that . **(6)**

   c) Verifying the hypothesis find the point 'c' satisfying mean value theorem for the function in [0 2]. **(6)**

3. a) State and prove second derivative test for extrimum. **(6)**

   b) State and prove Mean value theorem. **(6)**

   c) Find the absolute extrema of in [-2 3] **(6)**

UNIT-II

4. a) Find all the asymptotes of ———.

(6)

b) State and prove Cauchy's Mean Value Theorem.

(6)

c) Draw a sketch of the graph of .

(6)

5. a) Evaluate (i) lim(sin) (ii) lim

(6)

f) Let *f* and *g* be functions that are differentiable on an open interval I, except possibly at the number *a* in *I*. Suppose that for all Then if lim()0 and lim()0, and if ——. Prove that lim ——

(6)

g) Draw the graph of

UNIT-III

6. a) Derive reduction formula for

(6)

b) Find the exact value of the definite integral as a limit of Reimann sum with a regular partition of the interval [1, 3] and a suitable choice of . (6)

c) Estimate ——— using Trapeziodal rule by taking $n = 6$.

(6)

7. a) Evaluate

(6)

b) Derive a reduction formula for $\int \tan^n x \sec x$

(6)

c) Evaluate

(6)

UNIT-IV

8\. a) Find the length of the arc of the curve from the point (1, 2) to the point (27, 18)

(6)

b) The region bounded by the curve and the line is revolved about x axis to generate a solid. Find the volume of the solid.

(6)

c) Find the area of the region inside the circle and outside the limacon

(6)

9\. a) Find the length of the arc of the curve from the origin to the point

(6)

b) The lease of a solid is the region enclosed by an ellipse having the equation . Find the volume of the solid if all phrase sections perpendicular to the x-axis are squares.

(6)

c) Find the area of the region in the plane enclosed by the cardioids

(6)

UNIT-V

10\. a) Prove that linear diaphantine equation has a solution iff where . If is any particular solution, then prove that all other solutions are given by — — for an arbitrary integer f. **(6)**

b) For positive integers a and b prove that .

(6)

c) If a cock is worth 5 coins, a hen 3 coins and 3 chickens together 1 coin, how many cocks, hens and chickens totaling 100 can be bought for 100 coins. **(6)**

11. a) Given integers a and b not both zero prove that there exists integers x and y such that

(6)

b) Solve the linear Diophantine equation .

(6)

c) Given integers a and b with . Prove that there exists unique integers q and r such that

(6)

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MAT 101.2 **Reg. No. ....**

CREDIT BASED FIRST SEMESTER B.Sc. DEGREE EXAMINATION
OCTOBER 2015

MATHEMATICS

PAPER I: CALCULUS AND NUMBER THEORY

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A
3x10=30

1. a) Find the critical members of

b) Find the value of c satisfying Rolle's theorem for the function in the interval $[3,3]$.

pp) Find the absolute extrema of the function on $[3,1]$

qq) Find $\sqrt{}$

rr) Find the oblique asymptote of ———

ss) Convert the polar equation to Cartesian form.

tt) Evaluate

uu) Find the value of for the function — such that

.

vv) Derive the reduction formula for .

ww) Find the length of the curve – from $x = 0$ to $x = 3$.

.

xx) Find the area of the region bounded by the graph of .

yy) The region between √ and the x axis, is revolved about x axis to generate a solid. Find its volume.

zz) Prove that if a | c and b | c and then ab | c

aaa) Show that square of an integer leaves remainder 0 or 1 on division by 4.

bbb) If prove that gcd

PART - B

UNIT-I

2. a) Find relative extrema of the function using second derivative test. **(6)**

b) State and prove Mean value theorem. **(6)**

c) Determine the interval where is concave upwards or concave downwards, hence find points of inflection for f. **(6)**

3. a) Prove that if *f(x)* exists for all values of x in the open interval *(a, b)* and if f has a relative extremum at c, where a < c < b, then if exists, then **(6)**

b) Find all asymptotes of

(6)

c) Find a point c satisfying the mean value theorem for $f(x)$ in

(6)

UNIT-II

4. a) If and f and g are differentiable in an open interval I containing a and $g(x) \neq 0$ in I if $x \neq a$ prove that

(6)

b) Draw the graph of $f(x)$.

(6)

c) Find if it exists.

(6)

5. a) State and prove Cauchy's mean value theorem.

(6)

h) Derive Taylor polynomial of degree 3 for $f(x) = \cos x$ at

i) Sketch the graph of

UNIT-III

6. a) Derive the reduction formula for $\int \sec^n x\,dx$ and hence find $\int \sec x\,dx$

(6)

b) Evaluate $\int \sqrt{\;}\,dx$

(6)

c) Using Trapezoidal rule evaluate with n = 6.

(6)

7. a) Derive reduction formula for $\int \cos x \sin x$

(6)

b) If f is continuous on [a, b] and let g be a function such that for all x in [a, b], then prove that

(6)

c) Evaluate

(6)

UNIT-IV

8. a) Find the volume of the solid generated by revolving about the x axis the region bounded by the parabola and the line using circular disc method. **(6)**

b) If the base of the solid is the region enclosed by a circle with a radius of r units and if all plane sections perpendicular to a fixed diameter of the base are squares. Then find the volume of the solid. **(6)**

c) Find the length of the arc of the curve in the 1st quadrant from the point to $x = 1$. **(6)**

9. a) The region bounded by and the lines y = 1 and x = 2 is revolved about the line y = 3 to generate a solid. Find the volume of the solid by cylindrical shell method. **(6)**

b) Find the area of the region inside the circle and outside the limacon . **(6)**

c) If C is a curve defined by , , where f and are continuous on [a, b], then derive the expression for the length of the curve from x = a to x = b. **(6)**

UNIT-V

10. a) Prove that linear diaphantine equation has a solution if and only if where . If is any particular solution of this equation, then prove that all the other solutions are given by — and — for varying integer t. **(6)**

b) State and prove Division algorithm. **(6)**

c) Find g.c.d. of 172 and 20 and express it in the form $172x + 20y$ in two different ways.

(6)

11. a) Given integers a and b, not both of which are zero, then prove that there exist integers x and y such that [illegible].

(6)

b) Let a and b be integers not both zero. Then prove that a and b are relatively prime if and only if there exist integers x and y such that $ax + by = 1$.

(6)

c) A customer bought a dozen pieces of fruit, apples and oranges for $ 1.32. If an apple costs 3 cents more than an orange and more apples were purchased than oranges, how many pieces of each kind were bought?

(6)

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PART - B

UNIT-I

2. a) State and prove mean value theorem. (6)

b) For the function $f(x)=x^3-6x^2+9x+1$, find the points of inflection of the graph of f and determine where the graph is concave upward and where it is concave downward. (6)

c) Find relative extrema of the function $f(x)=\frac{1}{3}x^3-x^2+3$ (6)

3. a) State and prove second derivative test for relative extrema. (6)

b) Let f be a function that is differentiable on some open interval containing C, then

i) if $f''(c)>0$, the graph of f is concave upward at $(c, f(c))$

ii) if $f''(c)<0$, the graph of f is concave downward at $(c, f(c))$ (6)

c) Points A and B are opposite each other on shores of a straight river 3 km wide. Point C is on the same shore as B but 2 km down the river from B. A telephone company wishes to lay a cable from A to C. If the cost per kilometer of the cable s 25 percent more under water than it is on land, what line of cable would be least expensive for the company? (6)

UNIT-II

4. a) State and prove Cauchy's mean value theorem. (6)

b) Draw the graph of $f(x)=\frac{x^2}{x^2-4}$ (6)

c) Find Taylor polynomial of degree 4 for f(x)=cosx at $x=\frac{\pi}{4}$ (6)

5. a) Let f and g be functions that are differentiable on an open interval I, except possibly at the number a in I. Suppose that for all $x\neq a$ in I, $g'(x)\neq 0$. If $\lim_{x\to a} f(x)=0$, $\lim_{x\to a} g(x)=0$ *and* $\lim_{x\to a}\frac{f'(x)}{g'(x)}=L$, then prove that $\lim_{x\to a}\frac{f(x)}{g(x)}=L$ (6)

b) Evaluate $\lim_{x\to 0^+}(x+1)^{\cot x}$ if it exists. (6)

c) Sketch the graph of $r=1-2\cos\theta$ (6)

UNIT-III

6. a) Derive the reduction formula for $\int \cos^m x\sin^n x\, dx$. (6)

b) Evaluate $\int \frac{\sqrt{x}dx}{1+\sqrt[3]{x}}$ (6)

c) Estimate $\int_0^3 \frac{1}{9+x^2}dx$ using trapezoidal rule for $n=6$. (6)

7. a) Find the exact value of $\int_0^2 x^3 dx$ as a limit of Riemann sum with regular partitions and for suitable choice of ξ_i (6)

b) Evaluate $\int \tan^5 x\sec^6 x\, dx$. (6)

c) Derive a reduction formula for $\int x^m(\log x)^n dx$ and hence evaluate $\int_0^1 x^4(\log x)^2 dx$. (6)

UNIT-IV

8. a) Find the volume of the solid generated by revolving about the line x = 1 the region bounded by the curve $(x-1)^2=20-4y$ and the lines x = 1, y = 1 and y = 3, and to the right of x = 1. (6)

b) Find the length of the arc of the curve $y^3=8x^2$ from the point (1,2) to the point (27,18) (6)

c) Find the area of the region inside the circle $r=3\sin\theta$ and outside the limacon $r=2-\sin\theta$ (6)

9. a) The base of a solid is the region enclosed by a ellipse having the equation $3x^2+y^2=6$. Find the volume of the solid if all plane sections perpendicular to the x axis are squares. (6)

b) If C is a curve defined by $y=f(x)$ where f and f' are continuous on [a b], then prove that the length of the curve from x = a to x = b is given by $L=\int_a^b\sqrt{1+(f'(x))^2}dx$ (6)

c) Find the area of the region in the plane enclosed by the cardioid $r=2(1+\cos\theta)$. (6)

UNIT-V

10. a) Given integers a and b not both zero prove that there exist integers x and y such that gcd(a,b) = $ax + by$ (6)

b) State and prove Division Algorithm. (6)

c) If a cock is worth 5 coins, a hen 3 coins and 3 chicks together one coin, then how many cocks, hens and chicks totaling 100 can be bought for 100 coins. (6)

11. a) Let a and b be integers not both zero. Then prove that a and b are relatively prime if and only if there exist integers x and y such that $ax + by = 1$ (6)

b) If $a = bq + r$, then prove that $gcd(a, b) = gcd(b, r)$. (6)

c) Solve the linear Diophantine equation $172x + 20y = 1000$. (6)

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MAT 101.2 Reg. No. ....

CREDIT BASED FIRST SEMESTER B.Sc. DEGREE EXAMINATION OCTOBER 2016

MATHEMATICS

PAPER I: CALCULUS AND NUMBER THEORY

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.

2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A **3x10=30**

1. a) Prove that $f(x) = x^{2/3}$ has no relative extrema.

b) Find the critical numbers of $f(x) = x^4 + 4x^3 - 2x^2 - 12x$.

c) Find the value of c satisfying Rolle's theorem for the function $f(x) = \frac{x^3}{3} - 3x$ in the interval [-3, 3].

d) Find $\lim_{x \to -\infty} \frac{3x+4}{\sqrt{2x^2 - 5}}$

e) Find an oblique asymptote of $f(x) = \frac{x^2 - 8}{x - 3}$

f) Find a Cartesian equation of $r^2 = 4\sin 2\theta$.

g) Evaluate $\int_0^{\pi/2} \cos^8 x \, dx$

h) Find the value of χ satisfying mean value theorem for integration for the function $f(x) = 3 - \frac{3x}{2}$ in the interval [0, 2].

i) Evaluate $\int x^3 e^{2x} \, dx$.

j) Find the length of the curve $y = (x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$.

k) The region bounded by the curve $y = x^2$, the x axes and the line $x = 2$ is revolved about y axis. Find the volume of the solid generated.

l) Find the area of the region bounded by the graph of $r = 3\cos\theta$.

m) Prove that if a | c and b | c and gcd (a, b) = 1 then ab | c.

n) Find whether the equation $14x + 12y = 51$ has a solution or not.

o) Find lcm (3054, 12378) if gcd (3054, 12378) = 6.