MAT 401 **REG NO. ....**

CREDIT BASED FOURTH SEMESTER B. Sc. DEGREE EXAMINATION
APRIL 2012

MATHEMATICS - IV

MULTIPLE INTEGRALS, COMPLEX VARIABLES, ANALYTICAL GEOMETRY AND GROUP THEORY

Duration: 3 hours **Max marks: 120**

Note: Answer any TEN questions from Part–A. Each question in Part–A carries 3 marks.
Answer FIVE full questions from Part–B choosing ONE full question from each unit.

PART A 3x10=30

1. a) Evaluate $\int_1^4 \int_{y^2}^{y} \sqrt{\frac{y}{x}}\, dx\, dy$

b) Find by double integration the area of the region enclosed by r = cosθ and the lines θ = 0 and $\theta = \frac{\pi}{6}$

c) Evaluate the line integral $\int_c 3x\,dx + 2xy\,dy + z\,d$ where

C: $x = \cos t, y = \sin t,\ z = t, 0 \le t \le 2\pi$

d) Determine the singular points of the function $f(z) = \frac{z^3+i}{z(z^2-3z+2)}$

e) Show that $f'(z)$ does not exist anywhere for $f(z) = 2x + ixy^2$.

f) Show that $f(z) = (3x + y) + i(3y - x)$ is entire.

g) Given the equation $4x^2 - y^2 = 9$, find an equation of the graph with respect to the $\bar{x}$ and $\bar{y}$ axes after a rotation of axes through an angle of radian measure $\frac{\pi}{4}$

h) Find the angle of rotation required to remove the mixed term in the equation $4x^2 + 2xy + y^2 - 8x + 8y = 0$

i) Remove *xy* term from the equation $xy + 4 = 0$.

j) Show that every subgroup of an obelian group is normal.

k) If G is a group and $a \in G$, let $N(a) = \{x \in G \mid ax = xa\}$, then show that N(a) is a subgroup of G.

l) Prove that intersection of two subgroups of a group G is itself a subgroup of G.

m) Let G be a group of all positive real numbers under multiplication and $\bar{G}$ be the group of all real numbers under addition. Define $\phi : G \to \bar{G}$ by $\phi(x) = \log_{10} x$. Show that ϕ is a homomorphism and find the Kernel of ϕ.

n) Compute $a^{-1}ba$ if a = (1 3 5) (1 2), b = (1 5 7 9).

o) i) What is the order of the symmetric group S_n?

ii) What is the number of even permutations of *n* elements?

PART B
UNIT 1

2. a) Find an approximate value of the double integral $\iint_R (3x - 2y + 1)\,dA$ where R is the rectangular region having vertices (0, −2) and (3, 0). Take a partition of R formed by the lines x = 1, x =2 and y = −1. Take (ξ_i, η_i) at the centre of the i^{th} sub region. (6)

b) Find the volume of the solid under the plane z = 4x and above the circle $x^2 + y^2 = 4$ in the *xy* plane using double integration. (6)

c) Evaluate $\int_0^1 \int_0^x \int_0^{x+y} (x + y + z)\, dz\, dy\, dx$ (6)

3 a) Suppose that a particle moves along the parabola $y = x^2$ from the point (−1, 1) to the point (2, 4). Find the total work done if the motion is caused by the force field

$F(x, y) \square (x^2 \square y^2) i \square 3x^2 y\, j$. Assume that arc is measured in metres and the force is measured in Newtons. (6)

b) Find the surface area of the paraboloid $z = x^2 + y^2$ below the plane z = 4. (6)

c) Evaluate $\iint_R e^{-(x^2+y^2)}\, dA$ where the region R is in the first quadrant and bounded by the circle $x^2 + y^2 = a^2$ and the coordinate axes. (6)

UNIT 2

4 a) Suppose that $f(z) = u(x,y) + iv(x,y)$, $z_0 = x_0 + iy_0$ and $w_0 = u_0 + iv_0$, then prove that $\lim_{z \to z_0} f(z) = w_0$ if and only if $\lim_{(x,y)\to(x_0,y_0)} u(x,y) = u_0$ and $\lim_{(x,y)\to(x_0,y_0)} v(x,y) = v_0$. (6)

b) If $u(x, y) = y^3 - 3x^2 y$, then show that $u(x, y)$ is harmonic in some domain and find its harmonic conjugate. (6)

c) Show that $f(z) = z - \bar{z}$ is not differentiable at any point on C. (6)

5 a) Suppose that $f(z) = u(x,y) + iv(x,y)$ and that $f'(z_0)$ exists at $z_0 = x_0 + iy_0$. Prove that the first order partial derivatives exist and satisfy Cauchy Riemann equations. (5)

b) If $f(z) = \begin{cases} \dfrac{(\bar{z})^2}{z} & \text{when } z \neq 0 \\ 0 & \text{when } z \square 0 \end{cases}$ then show that $f'(0)$ does not exist. (6)

c) Using $\epsilon - \delta$ definition show that $\lim_{z \to 1} \frac{iz}{2} = \frac{i}{2}$ in the open disc | z | < 1 (6)

UNIT 3

6 a) If (x, y) represents a point P with respect to a given coordinate axes and $(\bar{x}, \bar{y})$ is a representation of P after the axes have been rotated through an angle α, then prove that $x = \bar{x} \cos \alpha - \bar{y} \sin \alpha$ and $y = \bar{x} \sin \alpha + \bar{y} \cos \alpha$ (9)

b) Remove *xy* term from the equation $24xy \square 7y^2 \square 36 \square 0$. (9)

7 a) Simplify the equation $x^2 + xy + y^2 - 3y - 6 = 0$ by rotation and translation of axes (9)

c) Remove *xy* term from the equation $31x^2 \square 10\sqrt{3}xy \square 21y^2 \square 144$. (9)

UNIT 4

8 a) Prove that there is a one to one correspondence between any two right cosets of H in G. (6)

b) If G is a finite group and H is a subgroup of G, then prove that O(H) is a divisor of O(G). (6)

d) Prove that every group of prime order is cyclic. (6)

9 a) Prove that the relation $a \equiv b \pmod{H}$ is an equivalence relation ($a \equiv b \pmod{H}$ if $ab^{-1} \in H$) (6)

b) Let H and K be two subgroups of G. Prove that HK is a subgroup of G if and only if HK = KH. (6)

c) Prove that a subgroup N of a group G is a normal subgroup of G if and only if the product of any two right cosets of N in G is again a right coset of N in G. (6)

UNIT 5

10 a) Let ϕ be a homomorphism of G onto [illegible] with kernel K, then prove that [illegible] (6)

b) Prove that S_n has as a normal subgroup of index 2, the alternating group A_n of even permilations. (6)

c) Express the permutation (1 2 3) (4 5) (1 6 7 8 9) (1 5) as a product of disjoint cycles. State whether it is even or odd. (6)

11 a) Prove that Kernel of a homomorphism is a normal subgroup (6)

b) Prove that set of all inner automorphisms of any group G forms a subgroup of the set of all automorphisms of G. (6)

c) Prove that every permutation is a product of its disjoint cycles. (6)

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MAT 401.1 **REG NO. ....**

CREDIT BASED FOURTH SEMESTER B. Sc. DEGREE EXAMINATION
APRIL 2013

MATHEMATICS - IV

MULTIPLE INTEGRALS, FUNCTIONS OF SEVERAL VARIABLES AND GROUP THEORY

Duration: 3 hours **Max marks: 120**

Note: Answer any TEN questions from Part–A. Each question in Part–A carries 3 marks. Answer FIVE full questions from Part–B choosing ONE full question from each unit.

PART A 3x10=30

11 $$\int_1^2 \int_0^{2x} xy^3 \, dy \, dx$$

1. a) Evaluate

b) Evaluate the double integral $\iint_R (2x^2 - 3y)\,dA$ if R is the region consisting of all points (x,y) for which $-1 \le x \le 2$ and $1 \le y \le 3$.

c) Find the area of the surface cut from the plane $2x + y + z =$ [illegible] by the planes $x = 0, x = 1, y = 0$ and $y =$ [illegible]

d) Evaluate $\int_0^{\pi/4}\int_2^4\int_0^1 r\, e^z\, dz\, dr\, d\theta.$

e) Evaluate $\int_0^1\int_0^x\int_0^{x+y} (x + y + z)\,dz\, dy\, dx$

f) Evaluate the line integral $\int_C 3x\,dx + 2xy\,dy + z\,dz$ if the curve C is the circular helix defined by $x = \cos t,\ y = \sin t,\ z = t,\ 0 \le t \le 2\pi$

g) Given $F(t) = \log t$ and $G(x,y) = x^2 +$ [illegible], find the composite function $F \circ G$ and its domain.

h) Define continuity of a function of n variables.

i) If $f(x,y) =$ [illegible], then find $f_y(x,y)$.

j) If G is a finite group and $a \in G$ then prove that $a^{O(G)} = e$

k) Prove that intersection of two subgroups of a group G is itself a Subgroup of G.

l) If G is a group, $a \in G$ and N(a) = $\{ x \in G : ax = xa\}$ then prove that N(a) is a subgroup of G.

m) Let G be the group of real numbers under addition and if $\phi: G \to G$ is defined by $\phi(x) = 13x$ then show that ϕ is a homomorphism, and find its Kernel.

n) Prove that every permutation is a product of its cycles.

o) If ϕ is a homomorphism of G into $\bar{G}$ then prove that $\phi(e) = \bar{e}$ the unit element of $\bar{G}$.

PART B

UNIT 1

2. a) Find an approximate volume of the solid bounded by the surface $f(x,y) = (2x^2 - 3y)$ and the planes $x = -1,\ x = 2, y = 1$ and $y =$ [illegible] above the xy plane, by taking partition of the region by the lines $x = 0,\ x = 1,\ y =$ [illegible] and the values of the function at the centres of each subregion. (6)

b) Find the volume of the solid in the first octant bounded by the cone z=r and the cylinder $r = 3\sin\theta$. (6)

c) Find the area of the surface that is cut from the cylinder $x^2 + z^2 = $ [illegible] by the planes $x = 0,\ x = 2,\ y = $ [illegible] and $y = $ [illegible]. (6)

3 a) Find the surface area of the paraboloid $z = x^2 + y^2$ below the plane z = 4. (6)

b) Find the volume of the solid bounded by the surface $f(x,y) = 4 - \frac{1}{9}x^2 - \frac{1}{16}y^2$, the planes $x = $ [illegible] and $y = $ [illegible] and the three coordinate planes. (6)

c) Evaluate $\iint_R Sinx\, dA$ R is the region bounded by the lines $y = 2x,\ y = $ [illegible] and $x = \pi$. (6)

UNIT 2

4 a) Evaluate $\iiint_S xy\sin(yz)\, dV$ if S is the rectangular parallelopiped bounded by the planes $x = \pi,\ y = \frac{1}{2}\pi$ and $z = \frac{1}{3}\pi$ and the coordinate planes. (6)

b) Find the volume of the solid bounded by the paraboloid $x^2 + y^2 + z = $ [illegible] and the plane $z = 8$. (6)

c) Suppose a particle moves along a parabola $y = x^2$ from the point (-1, 1) to the point (2, 4). Find the total work done, if the motion is caused by the force field $F(x,y) = (x^2 + y^2)i + 3x^2yj$ where the arc is measured in metres and the force in newtons (6)

5 a) Evaluate $\int_0^1 \int_0^{1-x} \int_{2y}^{4y^2} x\, dz\, dy\, dx$ (6)

b) Find the volume of the solid in the first octant bounded by the cylinder $x^2 + y^2 = $ [illegible] and the plane $z = x$. (6)

c) Evaluate the line integral over the curve $\int_C F \cdot dR;\ F(x,y) = y\ sinx i - cosx j$ C: the line segment from $\left(\frac{\pi}{2}, 0\right)$ to $(\pi, 1)$. (6)

UNIT 3

6 a) Let the function f be defined by $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$.

Show that f is not continuous at (0,0). (6)

b) If $u = \ln\sqrt{x^2 + y^2}$, $x = re^{s}$, $y = re^{-s}$ find $\frac{\partial u}{\partial r}$ and $\frac{\partial u}{\partial s}$ using chain rule. (6)

c) Find the equation of the tangent plane to the elliptic paraboloid $4x^2 + y^2 - 16z = 0$ at the point (2, 4, 2). Hence find the symmetric equations of the normal line. (6)

7 a) Given $f(x,y) = e^x \cos y + (\tan^{-1} x)\ln x$ find $D_{11}f(x,y)$ and $\frac{\delta^3 f}{\delta x \delta y^2}$ (6)

e) If $f(x,y) = 2x^4 + y^2 - x^2 - 2y$, then determine the relative extrema of f, if there are any. (6)

f) Prove that $\lim_{(x,y)\to(1,2)} 3x^2 + y = 5$ using $\epsilon - \delta$ definition. (6)

UNIT 4

8 a) State and prove Lagrange's theorem. (6)

b) Define normal subgroup. If H is a subgroup of G and N is a normal subgroup of G, then show that $H \cap N$ is a normal subgroup of H. (6)

c) For all $a \in G$ prove that $Ha = \{x \in G \mid a \equiv x \pmod H\}$ (6)

9 a) If H and K are finite subgroups of G of orders $O(H)$ and $O(K)$ respectively. Then prove that $O(HK) = \frac{O(H)\,O(K)}{O(H \cap K)}$. (6)

b) Prove that a subgroup N of G is a normal subgroup of G if and only if the product of two right cosets of N in G is again a right coset of N in G. (6)

c) Prove that HK is a subgroup of G if and only if HK = KH. (6)

UNIT 5

10 a) If ϕ is a homomorphism of a group G onto a group $\bar{G}$ with kernel K, then prove that $G/K \cong \bar{G}$. (6)

b) Prove that the set of all inner automorphisms of any group G forms a subgroup of the set of all automorphisms. (6)

c) Prove that Kernel of a homomorphism is a normal subgroup of G. (6)

11 a) Let G be a group, prove that the set of all automorphisms of G is also a group. (6)

b) Prove that S_n has as a normal subgroup of index 2 the alternating group A_n, consisting of all even permutations. (6)

c) i) Compute $aa^{-1}b$ where $a = (5\ 7\ 9)$, $b = (1\ 2\ 3)$.
ii) Express (1 2) (1 2 3) (1 2) as the product of disjoint cycles. (6)

MAT 401.1 **Reg. No. ....**

CREDIT BASED FOURTH SEMESTER B.Sc. DEGREE EXAMINATION APRIL 2014

MATHEMATICS

PAPER IV: MULTIPLE INTEGRALS FUNCTIONS OF SEVERAL VARIABLES AND GROUP THEORY

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A **3x10=30**

1. a) Evaluate $\iint xy\,dy\,dx$

b) Evaluate the double integral . If R is the region consisting of all points (x, y) for which and .

c) Find the double integration the area of the region enclosed by one leaf of the rose

d) Evaluate

e) Evaluate $\iiint dz\,dy\,dz$

f) Evaluate

g) Given and find the function and its domain.

h) Given find it is exists.

i) Given find .

j) If G is finite group and then prove that

k) Show that is a subgroup of a group G.

l) If G is a group and H is a subgroup of index 2 in G, then prove that H is a normal subgroup of G.

m) Compute where

123456789

n) Find cycles of the permutation:

o) If is a homomorphism of G into $\bar{G}$, then prove that , the unit element of $\bar{G}$.

PART - B

UNIT-I

2. a) Approximate the volume of the solid in the first octant sphere , the planes x = 3, y = 3 and the three coordinate planes. To find an approximate value of the double integral take a partition of the region in the xy plane formed by the lines x =1, x = 2, y = 1 and y = 2, and take at the centre of the subregion. **(6)**

b) Find the volume of the solid bounded by the surface , the planes x = 3 and y = 2, and the three coordinate planes. **(6)**

c) Find the area of the region inside the cardioid . **(6)**

3. a) Find the area of the surface cut from the cylinder by the planes x = o, x = 2, y = o and y = 3. **(6)**

b) Find the volume of the solid in the first octant bounded by the cone and the cylinder . **(6)**

c) Evaluate ; R is the region bounded by the lines and **(6)**

UNIT-II

4. a) Find the volume of the solid bounded by the cylinder , the plane and the *xy* plane. **(6)**

b) Evaluate $\iiint dV$ if S is the rectangular parallelepiped bounded by the planes and the coordinate planes. **(6)**

c) Evaluate **(6)**

5. a) Suppose a particle moves along a parabola $y = x^2$ from the point (-1, 1) to the point (2, 4). Find the total work done, if the motion is caused by the force field where the arc is measured in metres and the force in newtons. **(6)**

b) Find the mass of the solid hemisphere of radius 'a' meters if the volume density at any point is proportional to the distance of the point from the axis of the solid and is measured in kilograms per cubic meter. **(6)**

c) Find the volume of the solid bounded by the cylinder , the plane and the xy plane. **(6)**

UNIT-III

6. a) Given $f(0,0)$ **(6)**

Show that $f(0, y)$ for all y

b) Prove that a differentiable function of two variables is continuous. **(6)**

d) Given $u = \cos x, \sin y$ find — and — using the chain rule. **(6)**

7. a) Find symmetric equations of the tangent line the curve of intersection of the surfaces and at the point (3, -2, 2) **(6)**

b) If $f(x, y)$ determine the relative extreme of f if there are any. **(6)**

e) Given $f(x, y, z)$ find the vase of change of $f(x, y, z)$ at (1, -2, -1) in the direction of the vector **(6)**

UNIT-IV

8. a) If H is a subgroup of G and N is a normal subgroup of G then prove that $H \cap N$ is a normal subgroup of H. **(6)**

b) State and prove Lagrange's theorem. **(6)**

c) Prove that HK is a subgroup of G if and only of HK = KH. **(6)**

9. a) Prove that N is a normal subgroup of G if and only if gN
for every **(6)**

b) If G is a finite group whose order is a prime number p, then show that G is a cyclic group. **(6)**

c) For all at G, prove that **(6)**

UNIT-V

10. a) Prove that kernel of a homomorphism is a normal subgroup of G. **(6)**

b) Prove that set of all inner automorphism of any group G forms a subgroup of the set of all automorphism. **(6)**

c) Express (1 2) (1 2 3) (1 2) as the product of disjoint cycles. Find its order. Also determine whether it is even or odd.

11. a) Let be a homomorphism of G onto $\overline{G}$ with kernel K, then prove that
. **(6)**

b) Prove that every permutation is a product of disjoint cycles. **(6)**

c) Let G be the group of positive real number under multiplication and let $\overline{G}$ be the group of all real numbers under addition. Define by . Show that is a homomorphism. Find its kernel. Is it an isomorphism? **(6)**

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MAT 401.1 **Reg. No. ....**

CREDIT BASED FOURTH SEMESTER B.Sc. DEGREE EXAMINATION APRIL 2015

MATHEMATICS

PAPER IV: MULTIPLE INTEGRALS, FUNCTIONS OF SEVERAL VARIABLES AND GROUP THEORY

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A 3x10=30

1. a) Evaluate $\sqrt{\quad}$

b) Find the volume of the solid bounded by the surface

– — and co-ordinate planes.

p) Find the area of the surface cut from the plane by the planes and

q) Evaluate $\iiint xyz\,dz\,dy\,dx$

r) Evaluate

s) Evaluate

t) If $f(x,y)$ find domain of fog

u) Find the directional derivative of $f(x,y)$ in the direction of $u = \cos\frac{\pi}{4} i + \sin\frac{\pi}{4} j$

v) Find gradient of f at (4, 3) if $f(x,y) = 4x^3y$.

w) Prove that centre of a group G is a subgroup of G.

x) Prove that every subgroup of an abelian group is normal.

y) If G is a finite group and , then prove that

z) If G is defined by $f(x)$ where G is the group of nonzero real numbers with respect to miltiplication, prove that is a homomorphism, and find its kernel.

aa) If is a homomorphism of groups defined by then find where are groups with respect to + (mod 4) and x(mod 5) respectively.

bb) Find where $a = (1\ 3\ 5)(1\ 2)$ and b = (1 5 7 9)

PART - B

UNIT-I

2. a) Find by double integration, area of the region in xy plane bounded by the curves and **(6)**

b) Find the volume of the solid above the xy plane bounded by the elliptic paraboloid and the cylinder **(6)**

c) Find the area of the region inside the cardioid and outside the circle **(6)**

3. a) Find the volume of the solid in the 1st octant bounded by 2 cylinders and **(6)**

b) Find an approximate value of the double integral where R is rectangular region having vertices P (0, 2), Q (3, 0). Take the partition of R formed by lines x=1, x=2 and y= 1 **(6)**

c) Find the area of the top half of the sphere **(6)**

UNIT-II

4. a) Evaluate where C consists of the line segment from (3, 2) to (1,0) and 1st quadrant arc of the circle from (1, 0) to (0, 1) traversed in counter clockwise direction. **(6)**

b) Find the volume of the solid in the 1st octant bounded by the cylinder and the plane . **(6)**

c) Evaluate the line integral over the curve , where where C: the line segment from to **(6)**

5. a) Find the volume of the solid bounded by the cylinder and the plane and the *xy* plane. **(6)**

f) A homogeneous solid in the shape of a right circular cylinder has a radius of 2m and an altitude of 4m. Find the moment of inertia of the solid with respect to its axis. **(6)**

g) Evaluate **(6)**

UNIT-III

6. a) If (6)

find whether is continuous at (0, 0).

b) If $\sin^{-1}(3x + y), x = r^2 e^s, y = \sin rs$ find — and — (6)

c) Find the equation of the tangent plane and normal line to the surface at (2,4,6) (6)

7. a) If — — find and (6)

b) Find the symmetric equations of the tangent line to the curve of intersection of the surfaces and at (2,2,0) (6)

c) Find relative extrema of the function, if any, for (6)

UNIT-IV

8. a) If H is a nonempty finite subset of a group G and is closed under multiplication, prove that H is a subgroup of G. (6)

b) State and prove Lagrange's Theorem. (6)

c) If N is a normal subgroup of G and H is any subgroup of G, prove that NH is a subgroup of G. (6)

9. a) If H is a subgroup of G, prove that any two right cosets of H in G are either identical or disjoint (6)

b) If H and K are 2 subgroups of a group G, prove that (6)

c) Prove that a subgroup N of G is normal in G if and only if every left coset is equal to a right coset of N in G. (6)

UNIT-V

10. a) If is a homomorphism from G onto and K = Ker , prove that (6)

b) Define Automorphism of groups. Prove that defined by is an automorphism . (6)

c) Express the permutation (1 2 3) (4 5) (1 6 7 8 9) (1 8) as the product of disjoint cycles. Find its order. State whether it is even or odd. (6)

11. a) Prove that a homomorphism G into $\bar{G}$ is an isomorphism if and only if Ker $=\{e\}$. **(6)**

b) Prove that every group is isomorphic to a subgroup of A(S). **(6)**

c) Prove that the set A_n of all even permutations is a normal subgroup of S_n, the set of all permutations on *n* symbols. **(6)**

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c) Find $\int_C \bar{z}\, dz$ where C is the right-hand half $z = 2e^{i\theta}$, $\left(-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right)$ of the circle $|Z| = 2$ from $z = -2i$ to $z = 2i$ **(6)**

11. a) If $u(x,y) = y^3 - 3x^2 y$ then show that $u(x,y)$ is harmonic in some domain and find its harmonic conjugate. **(6)**

b) If m and n are integers, then find the value of $\int_0^{2\pi} e^{im\theta} \cdot e^{-in\theta} d\theta$ for $m = n$ and $m \ne n$ **(6)**

c) Evaluate $\int_C \frac{z+2}{z} dz$ where C is the semicircle $z = 2e^{i\theta}$, $(0 \le \theta \le \pi)$. **(6)**

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MAT 401.2 Reg. No. ....

CREDIT BASED FOURTH SEMESTER B.Sc. DEGREE EXAMINATION APRIL 2016

MATHEMATICS

PAPER IV: ANALYTICAL GEOMETRY, RING THEORY AND COMPLEX VARIABLES

Duration: 3 hours **Max Marks: 120**

Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.
2. Answer FIVE full questions from Part B choosing ONE full question from each unit.

PART A **3x10=30**

1. a) Find an equation of the graph of xy = 1 w.r. to $\bar{x}$ and $\bar{y}$ axes after a rotation of axes through an angle of radian measure $\frac{\pi}{4}$.

b) Find the eccentricity and directrices of the hyperbola $4x^2 - 25y^2 = 100$.

c) Find the eccentricity of the conic $r = \frac{6}{4 + 5\sin\theta}$ and identify the conic.

d) If p is a prime number then prove that J_p, the ring of integers mod p is a field.

e) If every element x in a ring R satisfies $x^2 = x$, prove that R is commutative.

f) If U is an ideal of R and $1 \varepsilon U$, prove that U = R.

g) In (Z, +, ·) prove that P=pZ (where p is a prime) is a prime ideal.

h) Find all units in J[i].

i) Prove that $f(x) = x^2 + x + 1$ is irreducible over the field of integers mod 2

j) Find the principal argument Argz when $z = \frac{-2}{1+\sqrt{3}i}$

k) Sketch the set $|2z+3| > 4$

l) i) Find the domain of definition of $f(z) = \frac{z}{z+2}$

ii) Show that $f'(z)$ does not exist anywhere for $f(z) = 2x + ixy^2$

m) Show that $f(z) = \bar{z}$ is not analytic anywhere.

n) Find the singular points of the function $f(z) = \frac{2z+1}{z(z^2+1)}$.

o) Show that the function $f(z) = (3x+y) + i(3y-2)$ is entire.

PART - B

UNIT-I

2. a) If $B \neq 0$ then prove that the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ can be transformed into the equation $\bar{A}\bar{x}^2 + \bar{C}\bar{y}^2 + \bar{D}\bar{x} + \bar{E}\bar{y} + \bar{F} = 0$ when $\bar{A}$ and $\bar{C}$ are not both zero, by a rotation of axes through an angle α for which $Cot2\alpha = \frac{A-C}{B}$ (6)

   b) Remove xy term from the equation $x^2 + 2xy + y^2 - 8x + 8y = 0$ by a rotation of axes. Draw a sketch of the graph and show both sets of axes. (6)

   c) An equation of a conic is $r = \frac{5}{2 + \sin\theta}$

   Find the eccentricity, identify the conic, write an equation of the directrix corresponding to the focus at the pole, find the vertices, and draw a sketch of the curve. (6)

3. a) If (x, y) represents a point P with respect to a given set of axes and $(\bar{x}, \bar{y})$ is representation of P after the axes have been rotated through an angle α then prove that $x = \bar{x}\cos\alpha - \bar{y}\sin\alpha$ and $y = \bar{x}\sin\alpha + \bar{y}\cos\alpha$ (6)

   b) Simply the equation $17x^2 - 12xy + 8y^2 - 80 = 0$ by a rotation of axes. Draw a sketch of the graph of the equation and show both sets of axes. (6)

   c) The polar axis and its extension are along the principal axis of a hyperbola having a focus at the pole. The corresponding directrix is to the left of the focus. If the hyperbola contains the point $\left(1, \frac{2}{3}\pi\right)$ and $e = 2$.

   Find (i) an equation of the hyperbola.
   (ii) the vertices
   (iii) the centre
   (iv) an equation of the directrix corresponding to the focus at the pole. (6)

UNIT-II

4. a) Prove that every finite integral domain is a field. (6)

   b) If U and V are ideals of a ring R, prove that $U + V = \{u + v \mid u \,\varepsilon\, U, v \,\varepsilon\, V\}$ is also an ideal of R. (6)

   c) If ϕ is a homomorphism of a ring R into a ring R^1, prove that kernel of ϕ is an ideal of R. (6)

5. a) If R is a commutative ring with unit element whose only ideals are (0) and R itself, prove that R is a field. (6)

   b) If U is an ideal of a ring R, prove that $r(U) = \{x \,\varepsilon\, R \mid xu = 0, \quad \forall u \,\varepsilon\, U)\}$ is also an ideal of R. (6)

   c) Prove that every field is an integral domain. Is the converse true? Justify. (6)

UNIT-III

6. a) Let R be the ring of integers and p be a prime number. Prove that P = (p) is a maximal ideal of R. (6)

   b) Let R be a Euclidean ring and $a, b \,\varepsilon\, R$. If $b \neq 0$ is not a unit in R, then prove that $d(a) < d(ab)$ (6)

   c) Let R be a Euclidean ring and $a, b \,\varepsilon\, R$. Then prove that any two greatest common divisors of a and b are associates. (6)

7. a) Let R be a Euclidean ring and let A be an ideal of R. Then prove that there exists an element $a_0 \,\varepsilon\, A$ such that $A = \{a_0 x \mid x \,\varepsilon\, R\}$ (6)

   b) If p is a prime number of the form $4n + 1$ then prove that $p = a^2 + b^2$ for some integers a and b. (6)

   c) If $f(x)$ and $g(x)$ are two nonzero elements of the polynomial ring $F(x)$, prove that $\deg(f(x) \cdot g(x)) = \deg f(x) + \deg g(x)$ (6)

UNIT-IV

8. a) Find all the values of $\left(-8 - 8\sqrt{3}i\right)^{\frac{1}{4}}$. (6)

   b) If $f(z) = u(x,y) + iv(x,y)$, $z = x + iy$, $z_0 = x_0 + iy_0$ and $w_0 = u_0 + iv_0$ then prove that $\lim_{z \to z_0} f(z) = w_0$ if and only if $\lim_{(x,y)\to(x_0,y_0)} u(x,y) = u_0$ and $\lim_{(x,y)\to(x_0,y_0)} v(x,y) = v_0$ (6)

   c) Using C.R equations show that $f'(x)$ exists for $f(z) = e^z$ (6)

9. a) Establish by $\varepsilon - \delta$ definition $\lim_{z \to 1} \frac{iz}{2} = \frac{i}{2}$ in the open disc $|z| <|$ (6)

   b) Suppose that $f(z) = u(x,y) + iv(x,y)$ and that $f'(z)$ exists at a point $z_0 = x_0 + iy_0$. Show that the first order partial derivatives of u and v must exist at $(x_0 y_0)$ and they satisfy the Cauchy Reimann equations $u_x = v_y$ and $u_y = -v_x$ at $(x_0 y_0)$ (6)

   c) If $f(z) = \frac{1}{z}$ then show that $f'(z) = -\frac{1}{z^2}$ where $z \neq 0$ by using the polar form of C.R equations. (6)

UNIT-V

10. a) If a function $f(z) = u(x,y) + iv(x,y)$ is analytic in a domain D, then prove that its components u and v are harmonic in D. (6)

    b) If $u(x,y) = 2x(1-y)$ show that $u(x,y)$ is harmonic in some domain and find its harmonic conjugate. (6)