**MAT 201.1** **REG NO. ........................................**

# CREDIT BASED SECOND SEMESTER B. Sc. DEGREE EXAMINATION

## APRIL 2012

## MATHEMATICS

## PAPER II: CALCULUS AND ANALYTICAL GEOMETRY

**Duration: 3 hours** **Max marks: 120**

**Note: Answer any TEN questions from Part–A. Each question in Part–A carries 3 marks. Answer FIVE full questions from Part–B choosing ONE full question from each unit.**

## PART A (3X10=30)

1. a) Find the average value of the function $f(x) = x^2$ in the interval [1, 3].

   b) Find $D_x \int_x^1 \sqrt{2+t}\, dt$

   c) Find the value of λ such that $\int_1^2 x^3\, dx = f(\lambda)(2-1)$.

   d) The region bounded by the curve y = secx, the x axis, the y axis and the line $x = \pi/4$ is revolved about x axis. Find the volume of the solid generated.

   e) Compute the length of the line segment of the line y = 3x from the point (1, 3) to (2, 6) using integration.

   f) Find the area of the region bounded by the graph of $r = 3\cos\theta$.

g) Find the angle between the lines joining the points (3, 1, 2), (4, 0, –4) and (4, –3, 3), (6, –2, 2).

h) Find the distance between the parallel lines $2x - 2y + z + 3 = 0$ and $4x - 4y + 2z + 5 = 0$.

i) Find the angle between the planes $2x + 4y - 6z - 11 = 0$ and $3x + 6y + 5z + 4 = 0$.

j) Find the equation of the tangent plane to the sphere $3(x^2 + y^2 + z^2) - 3x - 4y - 2z - 22 = 0$ at the point (2, 3, 1).

k) Prove that the line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z-4}{-1}$ is parallel to the plane $x - 2y - 4z + 7 = 0$.

l) Find the equation of the sphere which has its centre at (6, –1, 2) and touches the plane $2x - y + 2z - 2 = 0$.

m) Find an equation of the graph of $xy = 1$ with respect to the $\bar{x}$ and $\bar{y}$ axes after a rotation of axes through an angle of radian measure $\frac{\pi}{4}$ .

n) Find the angle through which the system has to be rotated to remove the xy term from the equation $x^2 + 2xy + y^2 - 8x + 8y - 8$

o) Identify the conic $r = \frac{3}{1-\cos\theta}$

## PART B
## UNIT 1

2. a) Find the exact value of the integral $\int_1^3 x^2\,dx$ as a limit of Reimann sum with a regular partition of the interval [1, 3] and suitable choice of $\xi_i$. (9)

b) Let the function be continuous on the closed interval [a ,b] and let x be a number in [a, b]. If F is the function defined by $F(x) = \int_a^x f(t)\,dt$ , then prove that $F'(X) = f(x)$. (9)

3 a) State and prove the mean value theorem for integrals. (9)

b) Find the area of the trapezoid that is the region bounded by the lines $x = 1$, $x = 3$, the $x$ axis and the line $2x + y = 8$ as the limit of a Reimann sum. Take inscribed rectangles. (9)

## UNIT 2

4 a) Find the length of the arc of the curve $y = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}}$ from the point where $x = 0$ to the point where $x = 3$. (5)

b) Find the area of the region inside the circle $r = 3sin\theta$ and outside the limacon $r = 2 - \sin\theta$ . (8)

c) Use cylindrical shell method to find the volume of the solid generated by revolving the region bounded by the curve $y = x^2$, the $x$ axis and the line $x = 2$ about the y axis. (5)

5 a) Find the volume of solid generated by revolving about the x – axis the region bounded by the parabola $y = x^2 + 1$ and the line $y = x + 3$. (6)

b) Find the area of the region bounded by the graph of $r = 2 + 2cos\theta$. (6)

c) If the base of a solid is the region enclosed by a circle with a radius of r units and if all plane sections perpendicular to a fixed diameter of the base are squares, then find the volume of the solid. (6)

## UNIT 3

6 a) If $\theta$ is the angle between two lines whose direction cosines are $l_1, m_1, n_1$ and $l_2, m_2, n_2$ respectively, then prove that $cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$. (6)

b) Show that the following points are coplanar and find the equation of the plane on which they lie: (0, –1, –1), (–4, 4, 4), (4, 5, 1) and (3, 9, 4). (6)

c) Find the equation of the plane through the points (9, 3, 6) and (2, 2, 1) and perpendicular to the plane $2x + 6y + 6z - 9 = 0$. (6)

7 a) A line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube. Prove that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$. (6)

f) Find the equation of the plane through the point (–10, 5, 4) and perpendicular to the line joining the points (4, –1, 2) and (–3, 2, 3). (6)

g) Find the equation of the plane which passes through the point (1–, 3, 2) and perpendicular to the two planes $x + 2y + 2z = 5$ and $3x + 3y + 2z = 8$. (6)

## UNIT 4

8 a) Find the symmetrical form of the equations of the line of intersection of the planes $x + 5y - z - 7 = 0$ and $2x - 5y + 3z + 1 = 0$. (6)

b) Find the equation of the sphere having the circle, $x^2 + y^2 + z^2 - 2x + 4y - 6z + 7 = 0$, $2x - y + 2z = 5$ as a great circle. (6)

c) Find the shortest distance between the lines $\frac{x-3}{-8} = \frac{y-0}{1} = \frac{z-3}{-1}$ and $\frac{x+13}{3} = \frac{y+17}{-2} = \frac{z-6}{-4}$. (6)

9 a) Find the image of the point (1, –2, 3) in the plane $2x - 3y + 2z + 3 = 0$. (6)

b) Show that the intersection of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z - 2 = 0$. and the plane $x + 2y + 2z = 20$ is a circle of radius $\sqrt{7}$. (6)

c) Prove that the lines $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$ and $\frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$ are coplanar. Find the point of intersection. (6)

## UNIT 5

10 a) If (x, y) represents a point P with respect to the given set of axes and $(\bar{x}, \bar{y})$ is a representation of p after the axes have been rotated through an angle α, then prove that $x = \bar{x}\cos\alpha - \bar{y}\sin\alpha$ and $y = \bar{x}\sin\alpha + \bar{y}\cos\alpha$. (9)

b) Simplify the equation $x^2 + xy + y^2 - 3y - 6 = 0$ by a rotation and translation of axes. Draw the sketch of the graph and show the 3 sets of axes. (9)

11. a) Find a polar equation of the ellipse having a focus at the pole and having vertices at (3, 0) and (1, $\pi$). (6)

b) Remove the $xy$ term from the equation $xy + 16 = 0$ by rotating the axes. (6)

c) An equation of a conic is $r = \frac{5}{3+2\sin\theta}$. Find the eccentricity, identify the conic, write an equation of the directrix corresponding to the focus at the pole, find the vertices and draw a sketch of the curve. (6)

********

**MAT 202** **REG NO. ........................................**

**CREDIT BASED SECOND SEMESTER B.C.A. DEGREE EXAMINATION**

**APRIL 2012**

# MATHEMATICS

**FUNDAMENTALS OF MATHEMATICS-2**

**Duration: 3 hours** **Max marks: 80**

**Note: Answer any SIX questions from Part–A. Each question carries 2 marks.**
**Answer FOUR full questions from Part–B choosing ONE full question from each unit.**

## PART A

**1.** **2x6=12**

a) Let A = {1, 2, 3, 4}, B={2, 3, 7}, C={1, 2, 5, 7}.
Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

b) Define equivalence relation. Give example.

c) If $\lceil x \rceil$ = the least integer greater than or equal to x, find $\lceil 3.33 \rceil$.

d) Show that the functions $f(x) = x^3$ and $g(x) = x^{1/3}$ for $x \in R$ are inverses of one another.

e) Write the following statement in symbolic form: "If either Jerry takes Calculus or Ken takes Sociology, then Larry will take English."

f) Construct the truth table for $(P \to Q) \wedge P$

g) Define a directed graph. Give example.

h) Define a binary tree. Give example.

## PART B

## UNIT 1

2 a) Define Cartesian product of two sets.
Let A = {$\alpha, \beta$} and B = {1, 2, 3}. Find $A \times B, B \times A, A \times A$ and $B \times B$ (4)

b) Given the relation matrices $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$ (5)

Find $M_{R \circ S}$, $M_{\tilde{R}}$, $M_{\tilde{S}}$ and show that $M_{\widetilde{R \circ S}} = M_{\tilde{S} \circ \tilde{R}}$

c) Let A be the set of factors of a particular positive integer m and let ≤ be the relation divides, i.e., $\leq = \{<x, y> \mid x \in A \wedge y \in A \wedge (x \text{ divides } y)\}$
Draw Hasse diagram for m = 30 and m = 45. (8)

3 a) Define subset. For any two sets A and B, show that
(i) A – B = A $\cap \sim$ B (ii) A $\subseteq B \Leftrightarrow \sim B \subseteq \sim A$ (5)

d) Let X = {1, 2, 3, 4} and
R = {<1, 1>, <1, 4>, <4, 1>, <4, 4>, <2, 2>, <2, 3>, <3, 2>, <3, 3> }.
Write the matrix of R and sketch its graph (6)

e) Find the maximal compatibility blocks of the following relation and also draw the graph. (6)

| | | | | |
|---|---|---|---|---|
| 2 | 1 | | | |
| 3 | 1 | 1 | | |
| 5 | 0 | 0 | 1 | |
| 6 | 1 | 0 | 1 | 1 |
| | 1 | 2 | 3 | 5 |

## UNIT 2

4 a) Let $f : R \to R$ and $g : R \to R$, where R is the set of real numbers. Find *fog* and *gof* where $f(x) = x^2 \quad 2$ and $g(x) = x + 4$. (6)

b) Let $*$ be a binary operation on X which is associative and which has the identity $e \in X$. If an element $a \in X$ is invertible, then prove that both its left and right inverses are equal. (6)

c) Define characteristic function of a set. Show that $\sim \sim A = A$. (5)

5 a) Let X = {1, 2, 3} and *f, g, h* and *s* be functions from X to X given by

f = {<1, 2>, <2, 3>, <3, 1>} g = {<1, 2>, <2, 1>, <3, 3>}

h = {<1, 1>, <2, 2>, <3, 1>} s = {<1, 1>, <2, 2>, <3, 3>}

Find *fog, sog and fohog.* (6)

b) Define one-to-one function and onto function.
If $f : R \to R$ be given by $f(x) = x^2 - 2$, then find $f^{-1}$. (5)

f) Let $g : I \times I \to I$, where I is the set of integers and $g(x, y) = x * y = x + y - xy$. Show that the binary operation is commutative and associative. Find the identify element. (6)

## UNIT 3

6 a) Determine whether the following is a tautology, contradiction or neither.

(i) $((\neg Q \wedge \neg P) \wedge Q)$

(ii) $(P \to (P \vee Q))$

(iii) $(P \rightleftarrows Q) \rightleftarrows ((P \wedge Q) \vee (\neg P \wedge \neg Q))$ (6)

b) Show the following equivalences

i) $P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$

(ii) $\neg(P \rightleftarrows Q) \Leftrightarrow (P \vee Q) \vee \neg(P \wedge Q)$ (6)

c) Construct the truth table for

$\neg(P \vee (Q \wedge R)) \rightleftarrows ((P \vee Q) \wedge (P \vee R))$. (5)

7 a) Write the truth table for the following:
(i) Disjunction (ii) conditional (iii) Biconditional (6)

h) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \vee R) \Leftrightarrow R$ (5)

i) Given the truth values of P and Q as T and those of R and S as F, find the truth values of the following:
(i) $(P \rightleftarrows R) \wedge (\neg Q \rightarrow S)$ (ii) $(P \vee (Q \rightarrow (R \wedge \neg P)) \rightleftarrows (Q \vee \neg S)$ (6)

## UNIT 4

8 a) Define the following
(i) Adjacent nodes (ii) loop (iii) Path (iv) Leaf (v) Parallel edges (5)

b) Define adjacency matrix of the graph G.
Let S = {a, b, c, d} and let R be the relation on S represented by the matrix.

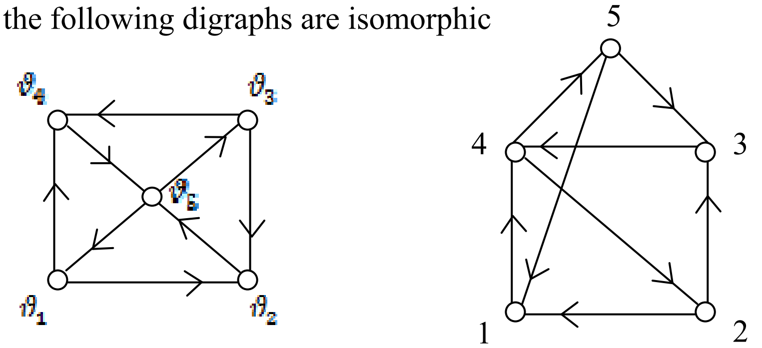
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Construct the digraph of R. Find $A \cdot A^T$ and $A^2$. (6)

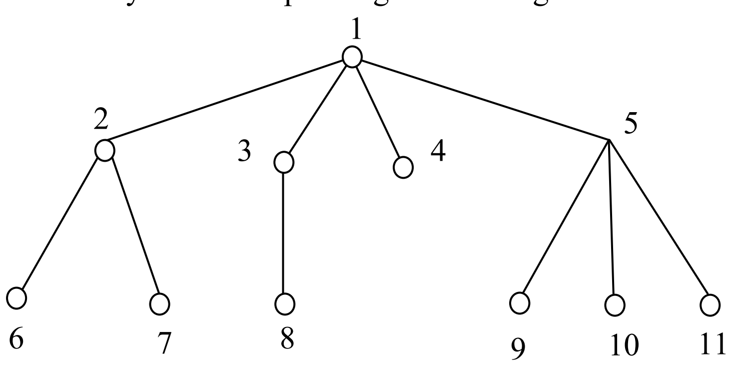
c) In a simple digraph G=<V,E>, prove that every node of the digraph lies in exactly one strong component. (6)

9 a) Define the following with an example.
i) Isolated node ii) Multigraph iii) Cycle (6)

b) Show that the following digraphs are isomorphic (6)




c) Obtain the binary tree corresponding to the tree given below. (5)



********

**MAT 201.1** **REG NO. ..........................................**

**CREDIT BASED SECOND SEMESTER B. Sc. DEGREE EXAMINATION**
**APRIL 2013**
## MATHEMATICS
**PAPER II: CALCULUS AND ANALYTICAL GEOMETRY**

**Duration: 3 hours** **Max marks: 120**

**Note: Answer any TEN questions from Part–A. Each question in Part–A carries 3 marks. Answer FIVE full questions from Part–B choosing ONE full question from each unit.**

**PART A** (3X10=30)

1. a) Find $D_x \int_x^1 \sqrt{2+t}\, dt$

   b) Find the average value of the function $f(x) = x^2$ in the interval [1, 2].

   c) Find the value of λ such that $\int_1^2 x^3 dx = f(\lambda)(2-1)$.

   d) Find the length of the arc of the curve $y = x^{2/3}$ from the point (1, 1) to the point (8, 4).

   e) Find the area of the region bounded by the graph of $r = 3\, sin\theta$

   f) Find the volume of the solid generated by revolving about the x – axis the region bounded by the curve $y = x^3$, the x – axis and the line x = 2.

   g) Find the angle between the planes $x - 2y - z = 0$ and $2x - y + z = 2$

   h) Find the distance of the origin from the plane $6x - 3y + 2z - 14 = 0$.

   ii) If $cos\alpha, cos\beta, cos\gamma$ are the direction cosines of a straight line, then show that $sin^2\alpha + sin^2\beta + sin^2\gamma = 2$.

   j) Find the distance between the parallel planes $2x + 2y - z + 6 = 0$ and $2x + 2y - z = 0$.

   k) Find the equation of the tangent plane to the sphere $3(x^2 + y^2 + z^2) - 2x - 4y - 3z - 22 = 0$ at the point (2, 3, 1)

   l) Show that the spheres $x^2 + y^2 + z^2 = 25$ and $x^2 + y^2 + z^2 - 18x - 24y - 40z + 225 = 0$ touch each other.

   m) Find the angle through which the system has to be rotated to remove the xy term from the equation $x^2 + xy + y^2 - 3y - 6 = 0$

   n) Identify the conic $r = \frac{3}{1 - cos\theta}$

   o) Find an equation of the graph of $x^2 - y^2 = 8$ with respect to the $\bar{x}$ and $\bar{y}$ axes after a rotation of axes through an angle of radian measure $\frac{\pi}{4}$ .

**PART B**
**UNIT 1**

2. a) Find the exact value of the definite integral $\int_1^3 x^2\,dx$ as a limit of Reimann sum with a regular partition of the interval [1, 3] and a suitable choice of $\xi_i$. (9)

b) Let the function $f$ be continuous on the closed interval [a ,b] and let $x$ be a number in [a, b]. If F is the function defined by $F(x) = \int_a^x f(t)\,dt$ , then prove that $F'(X) = f(x)$. (9)

3 a) State and prove the mean value theorem for integrals. (9)

b) Find the area of the region bounded by the curve $y = x^2$, the $x$ - axis and the line $x = 3$ by taking circumscribed rectangles. (9)

**UNIT 2**

4 a) If the function f and its derivative $f'$ are continuous on the closed interval [a, b] then prove that the length of the arc of the curve $y = f(x)$ from the point $(a, f(a))$ to the point $(b, f(b))$ is $L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$ (9)

b) The region bounded by the curve $y = x^2$and the lines y=1 and $x = 2$ is revolved about the line y = -3. Find the volume of the solid generated by taking the rectangular elements of area parallel to the axis of revolution. (9)

5 a) Find the volume of the solid generated by revolving about the x – axis the region bounded by the parabola $y = x^2 + 1$ and the line $y = x + 3$. (6)

b) Find the area of the region bounded by the graph of $r = 2 + 2\cos\theta$. (6)

c) The base of a solid is the region enclosed by an ellipse having the equation $3x^2 + y^2 = 6$. Find the volume of the solid if all plane sections perpendicular to the x – axis are squares. (6)

**UNIT 3**

6 a) If θ is the angle between two lines whose direction cosines are $l_1, m_1, n_1$ and $l_2, m_2, n_2$ respectively, then prove that $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$. (6)

b) Find the equation of the plane through the points (2, 2, –1), (3, 4, 2) and (7, 0, 6) (6)

c) Find the equation of the plane passing through the points (3, 1, 2) and (3, 4, 4) and perpendicular to the plane $5x + y + 4z = 0$. (6)

7 a) A line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube. Prove that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$. (6)

j) Find the equation of the plane which passes through the point (–1, 3, 2) and perpendicular to the planes $x + 2y + 2z = 5$ and $3x + 3y + 2z = 8$. . (6)

k) Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z - 7 = 0$ and perpendicular to the plane $x - 5y + 3z = 5$. (6)

**UNIT 4**

8 a) Find the perpendicular distance from P(3, 9, –1) to the line $\frac{x+8}{-8} = \frac{y-31}{1} = \frac{z-13}{5}$. (6)

b) Prove that the lines $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$ and $\frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$ are coplanar. Also find their point of intersection and the plane through them. (6)

d) Find the equation of the sphere having the circle, $x^2 + y^2 + z^2 - 2x + 4y - 6z + 7 = 0,\ 2x - y + 2z = 5$ as a great circle. (6)

9 a) Show that the plane $2x - y - 2z - 16$ touches the sphere $x^2 + y^2 + z^2 - 4x + 2y + 2z - 3 = 0$ and find the point of contact. (6)

b) Find the shortest distance between the lines $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$ and $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$. Also find the equation of the line of shortest distance. (6)

c) Find the image of the point (1, –2, 3) in the plane $2x - 3y + 2z + 3 = 0$. (6)

**UNIT 5**

10 a) Simplify the equation $17x^2 - 12xy + 8y^2 - 80 = 0$ by a rotation of axes. Also draw the sketch of the graph of the equation. (9)

b) If (x, y) represents a point P with respect to the given set of axes and $(\bar{x}, \bar{y})$ is a representation of P after the axes have been rotated through an angle α, then prove that $x = \bar{x}\cos\alpha - \bar{y}\sin\alpha$ and $y = \bar{x}\sin\alpha + \bar{y}\cos\alpha$ (9)

11. a) Remove the *xy* term from the equation $x^2 + 2xy + y^2 - 8x + 8y = 0$ by rotating the axes. (6)

b) An equation of a conic is $r = \frac{6}{4+5\sin\theta}$. Find the eccentricity, identify the conic, write an equation of the directrix corresponding to the focus at the pole, find the vertices and draw a sketch of the curve. (6)

c) Find a polar equation of the ellipse having a focus at the pole and having vertices at (3, 0) and (1, $\pi$). (6)

********

**MAT 202** **REG NO. .........................................**

**CREDIT BASED SECOND SEMESTER B.C.A. DEGREE EXAMINATION**
**APRIL 2013**

## MATHEMATICS

**FUNDAMENTALS OF MATHEMATICS-2**

**Duration: 3 hours** **Max marks: 80**

**Note: Answer any SIX questions from Part–A. Each question carries 2 marks.**
**Answer FOUR full questions from Part–B choosing ONE full question from each unit.**

### PART A

**1.** **2x6=12**

a) Represent the following using Venn diagram.
(i) A – B (ii) ~A

b) Define power set with example.

c) Let $f(x) = x + 2$, $g(x) = x - 2$ find *fog*.

d) If the function *f* is defined by $f(x) = x^2 + 1$ on the set {–2, –1, 0, 1, 2}. Find the range and domain of *f*.

e) Define negation of statement. Write the negation of "Bangalore is a city of Gardens".

f) Determine whether the statement $P \wedge \neg P$ is a tautology or a contradiction.

g) Define cyclic and acyclic graphs.

h) Define Indegree and Outdegree of a node.

### PART B
### UNIT 1

2 a) Let A = $\{x/x$ is an integer and $0 \leq x \leq 5\}$ , B = {3, 4, 5, 17} and C = {1, 2, 3}
Find (i) $A \cap B$ (ii) $A \cup B$ (iii) $A \cup C$ (iv) A – C (v) A – B (5)

g) Let X = {1, 2, 3, 4, 5, 6, 7} and R = {(x,y) | x–y is divisible by 3}. Show that R is an equivalence relation. Draw the graph of R. (6)

c) Given the relation matrices $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$.

Find $M_{RoS}$, $M_R$ , $M_S$ and $M_{RoS}$ and show that $M_{RoS} = M_{SoR}$ (6)

3 a) Let A = {1}, B = {a, b} and C = {2, 3} Find

(i) $B^2$ (ii) $B^2 \times A$ (iii) $A \times B$ (iv) $A \times B \times C$ (5)

b) Let X = {1, 2, 3, 4} and

R = {(1, 1), (1, 4), (4, 1), (4, 4), (2, 2), (2, 3), (3, 2), (3, 3) }.

Write the matrix of R and sketch its graph (6)

c) Find the maximal compatibility blocks of the following relation and also draw the graph. (6)

| | | | | |
|---|---|---|---|---|
| 2 | 0 | | | |
| 3 | 1 | 1 | | |
| 4 | 1 | 0 | 1 | |
| 5 | 0 | 1 | 0 | 1 |
| | 1 | 2 | 3 | 4 |

## UNIT 2

4 a) Let $*$ be a binary operation on X which is associative and which has the identity $e \in X$. If an element $a \in X$ is invertible, then prove that both its left and right inverses are equal. (5)

b) Let $f(x) = x + 2$, $g(x) = x - 2$ and $h(x) = 3x$ for $x \in R$, where R is the set of real numbers. Find *fog, fof, goh, fogog* and *fohog.* (6)

c) Define one-to-one function and onto function.
If $f : R \to R$ be given by $f(x) = x^2 - 2$, then find $f^{-1}$. (6)

5 a) Let A = {1, 2, 3} and *f, g are* functions from A to A given by

f = {(1,2),(2, 2), (3, 4), (4, 1)} g = {(1, 3), (2, 1), (3, 3), (4, 3)}

Find *fog, gof and gog.* (6)

b) Let $g : Z \times Z \to Z$, where Z is he set of integers and $g(x, y) = x * y = x + y - xy$. Show that the binary operation is commutative and associative. Find the identity element. (6)

c) Define characteristic function of a set. Show that $\sim\sim A = A$. (5)

## UNIT 3

6 a) Prove that $[(P \to Q) \wedge (Q \to R)] \to [(P \wedge Q) \to R]$ is a tautology. (6)

b) Write truth table of negation, conditional and biconditional statements. (5)

c) Prove the following:

(i) $(P \to Q) \Leftrightarrow (\neg Q \to \neg P)$

(ii) $P \wedge (P \rightarrow Q) \Rightarrow Q$ (6)

7 a) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ (6)

b) Determine whether the following is a tautology, contradiction or neither.

(i) $((\neg Q \wedge P) \wedge Q)$

(ii) $(P \rightarrow (P \vee Q))$

(iii) $(P \Leftrightarrow Q) \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$ (6)

c) Construct the truth table for the following statement.

$\neg (P \vee (Q \wedge R)) \Leftrightarrow ((P \vee Q) \wedge (P \vee R))$. (5)

## UNIT 4

8 a) Define the following with an example

(ii) Isomorphic graphs (ii) Weighted graphs (iii) Directed graphs

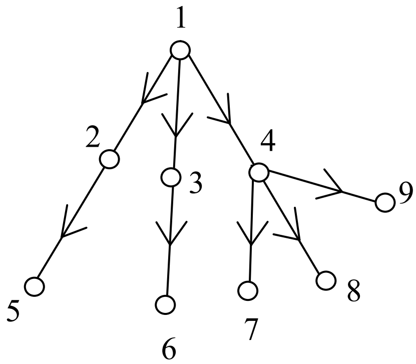
(iv) Multi graph (v) Null graph (10)

b) In a simple digraph $G = (V, E)$ Prove that every node of the digraph lies in exactly one strong component. (7)

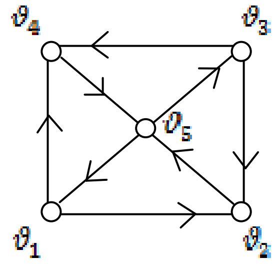
9 a) Define the following with an example

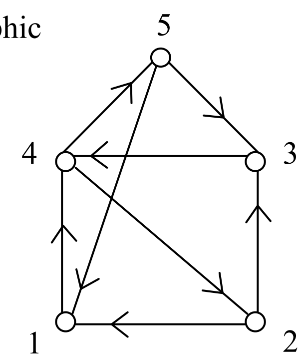
(i) Adjacent vertices (ii) loop (iii) Parallel edges (6)

b) Convert the following tree into a binary tree. (5)




b) Show that the following digraphs are isomorphic (6)



********

**MAT 201.1** **Reg. No. ..............................................**

**CREDIT BASED SECOND SEMESTER B.Sc. DEGREE EXAMINATION APRIL 2014**

**MATHEMATICS**

**PAPER II: CALCULUS AND ANALYTICAL THEORY**

**Duration: 3 hours** **Max Marks: 120**

**Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.**
**2. Answer FIVE full questions from Part B choosing ONE full question from each unit.**

**PART A** **3x10=30**

**1.** a) Find the average value of the function $f(x) = x^2$

b) Find the value of $\chi$ satisfying the mean value theorem for integral for $\int_1^2 x^3 dx$

c) Find $Dx \int_1^x \sqrt{2+t}\, dt$

d) Find the length of the arc of the curve $y = x^{2/3}$ from the point (1, 1) to the point (8, 4).

e) Find the volume of the solid generated, the region bounded by the curve $y = \sec x$, the $x$ axis, the y axis and the line $x = \pi/4$ revolved about $x$ axis.

f) Find the area of the region enclosed by the graph $r = 3\cos\theta$.

g) Find the direction cosines of the line joining the points (3, -5, 4) and (1, -8, -2).

h) Find the angle between the planes $2x + 4y - 6z - 11 = 0$ and $3x + 6y + 52 + 4 = 0$

i) If $\alpha, \beta, \gamma$ are the angles made of a straight line with the co-ordinate axes, then prove that $\sum \sin^2 2 = 2$ .

j) Find the equation of the tangent plane to the sphere $3\left(x^2 + y^2 + z^2\right) - 3x - 4y - 2z - 22 = 0$ at the point (2, 3, 1)

k) Find the equation of the plane passing through the points (0, 0, 0), (a, 0, 0), (0, b, 0) and (0, 0, c)

l) Prove that the plane section of a sphere is a circle.

m) Find an equation of the graph $xy = 8$ with respect to the $\bar{x}$ and $\bar{y}$ axes after a rotation of axes through an angle of radian measure $\frac{\pi}{4}$.

n) Identify the conic $r = \dfrac{5}{4 - \cos\theta}$

o) Find the angle through which the system has to be rotated to remove the $xy$ term from the equation $x^2 + xy + y^2 - 3y - 6 = 0$

## PART - B

### UNIT-I

2. a) Find the area of the region bounded by the curve $y = x^2$, $x$ axis and the line $x = 3$ by taking circumscribed rectangles. **(9)**

b) Let the function $f$ be continuous on the closed internal [a, b] and let x be a number in [a, b]. If F is the function defined by $F(x) = \int_a^x f(t)dt$, then prove that $F^1(x) = f(x)$ **(9)**

3. a) Find the exact value of the integral $\int_1^5 (2x-1)dx$ as a limit of Riemann sum with a regular partition of the interval [1, 5] and suitable choice of $\varepsilon_i$ **(9)**

b) State and prove the mean value theorem for integrals. **(9)**

### UNIT-II

4. a) Find the length of the arc of the curve $9y^2 = ex^3$ from the origin to the point $\left(3, 2\sqrt{3}\right)$ **(6)**

b) Find the area of the region bounded by the graph of $r = 2 + 2\cos\theta$. **(6)**

c) Use cylindrical shell method to find the volume of the solid generated by revolving the region bounded by the curve $y = x^2$, the $x$ axis and the line $x = 2$ about the $y$ axis. **(6)**

5. a) If the function f and its derivative $f^1$ are continuous on [9, 5], then prove that the length of the are of the curve $y = f(x)$ from $\left(a, f(a)\right)$ to $\left(b, f(b)\right)$ is $L = \int_a^b \sqrt{1 + \left(f^1(x)\right)^2}\, dx$ **(9)**

b) Find the area of the region inside the circle $r = 3\sin\theta$ and outside the limacon $r = 2 - \sin\theta$ **(9)**

## UNIT-III

6. a) Find the angle between two diagonals of a cube. **(6)**

   b) Find the equation of the plane which passes through the point (-1, 3, 2) and perpendicular to the planes $x+2y+2z=5$ and $3x+3y+2z=8$ **(6)**

   c) Show that the points (0, -1, -1), (-4, 4, 4), (4, 5, 1) and (3, 9, 4) are co-plannar. Also find the equation of the plane on which the above points lie. **(6)**

7. a) A line makes angle $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube. Prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta = 4/3$ **(6)**

   b) Find the equation of the plane through the points (9, 3, 6) and (2, 2, 1) and perpendicular to the plane $2x+6y+6z-9=0$ **(6)**

   c) Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z-7=0$ and perpendicular to the plan $x+5y+3z=5$ **(6)**

## UNIT-IV

8. a) Find in the symmetrical from the equation of line of intersection of the planes $x+5y-7-7=0$, $2x+5y+3z+1=0$ **(6)**

   b) Find the perpendicular distance from (3, 9, -1) to the line $\frac{x+8}{-8}=\frac{y-31}{1}=\frac{z-13}{5}$ **(6)**

   c) Find the equation of the sphere having the circle $x^2+y^2+z^2-2x+4y-6z+7=0,\ 2x-y+2z=5$ as a great circle. **(6)**

9. a) Find the image of the point (1, -2, 3) in the plane $2x-3y+2z+3=0$ **(6)**

   b) Find the shortest distance between the lines

   $$\frac{x-3}{-3}=\frac{y-8}{1}=\frac{z-3}{-1} \text{ and } \frac{x+3}{3}=\frac{y+7}{-2}=\frac{2-6}{-4}$$ **(6 )**

   c) Show that the intersection of the sphere $x^2+y^2+z^2-2x+4y-6z+2=0$, and the plane $x+2y+2z=20$ is a circle of radius $\sqrt{7}$ and centre at (2, 4, 5) **(6)**

## UNIT-V

10. a) If (x, y) represents a point P with respect to the given set of axes and $(\bar{x}, \bar{y})$ is a representation of P after the axes have been rotated through an angle $\alpha$, then prove that $x = \bar{x}\cos\alpha - \bar{y}\sin\alpha$ and $y = \bar{x}\sin\alpha - \bar{y}\cos\alpha$. **(9)**

b) Simplify the equation $17x^2 - 12xy + 8y^2 - 80 = 0$ by a rotation of axes. Also draw the sketch of the graph of the equation. **(9)**

11. a) Remove the $xy$ term from the equation $x^2 + 2xy + y^2 - 8x + 8y = 0$ by rotating the axes. **(6)**

b) An equation of a conic is $r = \dfrac{5}{3 + 2\sin\theta}$. Find the eccentricity. Identify the conic, write an equation of the direction corresponding to the focus at the pole, find the vertices and draw a sketch of the curve. **(6)**

c) For the given equation $x^2 - y^2 = 8$, find the equation of the graph with respect to the $\bar{x}$ and $\bar{y}$ axes after a rotation of axes through an angle of radian measure II. **(6)**

********************

**MAT 202** **Reg. No. ...............................................**

**CREDIT BASED SECOND SEMESTER B.C.A. DEGREE EXAMINATION APRIL 2014**

**MATHEMATICS**

**PAPER II: FUNDAMENTALS OF MATHEMATICS**

**Duration: 3 hours** **Max Marks: 80**

**Note: 1. Answer any SIX questions in Part A. Each question carries 2 marks.**
**2. Answer any FOUR full questions from Part B choosing ONE full question from each unit.**

**PART A** **2x6=12**

**1.** a) Let A = {1, 2, 3}, B = {2, 3} and C = {1, 3}
prove that $(A \cap B) \quad UC = A \cap (B \cup C)$

b) Define power set of a set A. If A has n elements what is the number of elements in P(A).

p) Define function with an example.

q) Find $f^{-1}$ if $f = x^3 - 4$.

r) Find the truth values of $P \vee (Q \wedge R)$ if the truth values of P.2 are T and that of R is F.

s) Construct the truth table for $(\neg P \vee Q)$.

t) Define binary tree with an example.

u) What is indegree and out degree of a node in a digraph.

## PART - B

### UNIT-I

2. a) If A = {1}, B = {a, b} and C = {2, 3} find
(i) $B^2$ (ii) $B^2 \times A$ (iii) $A \times B$ (iv) $A \times B \times C$ **(6)**

b) Define equivalence relation on a set.
If A = {1, 2,3, 4} write the following
(i) An equivalence relation on A
(ii) A relation which is not an equivalence relation non A. **(6)**

c) Define compatibility relative. Find the maximum compatibility blocks of the following relation. Also draw the corresponding graph. **(5)**

| | | | | |
|---|---|---|---|---|
| 2 | 0 | | | |
| 3 | 1 | 1 | | |
| 4 | 1 | 0 | 1 | |
| 5 | 0 | 1 | 0 | 1 |
| | 1 | 2 | 3 | 4 |

3. a) Define subset. For any two sets A and B show that
(i) $A - B \Leftrightarrow A \cap$ □ $B$ (ii) $A \leq B \Leftrightarrow$ □ $B \leq$ □ $A$ **(6)**

b) Define the following
(i) Union of two sets (ii) Disjoint sets (iii) Universal set
(iv) Compliment of a set (v) intersection of two sets **(6)**

c) Let A be the set of factors of a particular positive integer m and let $\leq$ be the relative divides. That is $\leq = \{< x, y > / x \, \varepsilon \, A \wedge y \, \varepsilon \, A \wedge (x \text{ divides } y)\}$
Draw Harse diagrams for m = 12 and m = 45. **(5)**

### UNIT-II

**4.** a) Show that the function $f : R_+ \rightarrow R$ defined by $f(x) = 2x^2 + 1$ is both one one and onto. Where $R_+$ is the set of positive real numbers and R is the set of real numbers. **(5)**

b) If f, g, h are three functions defined on the set of real numbers by $f(x) = x + 2, g(x) = x - 2$ and $h(x) = 3x$ find the following
(i) fog (ii) goh (iii) fogoh (iv) fogog **(6)**

c) Define characteristic function of a set show that □□ $A = A$. **(6)**

5. a) Let f and g be two functions on the set of real numbers defined by $f(x) = x^2 - 2$ and $g(x) = x + 4$. State whether these functions are injective, surjective and bijective.
**(6)**

d) Define identity element under a binary operation if $e_e$ is the left identity element and $e_r$ right identity element with respect to then prove that $e_e = e_r$ **(5)**

e) Using the properties of characteristic functions prove that
$A \cap (B \cup C) = A(\cap B) \cup (A \cap C)$ **(6)**

## UNIT-III

6. a) Construct truth table for the formula $7(P \vee (Q \wedge R))$□ $(P \vee Q) \wedge (P \vee R)$ **(6)**

b) Show the implication $(P \to (Q \to R)) \Rightarrow (P \to Q) \to (P \to R)$ **(6)**

c) Prove the following equivalence
(i) $P \to (Q \to P) \Leftrightarrow 7P \to (P \to Q)$
(ii) $P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$ **(6)**

7. a) Translate the following statements in symbolic form. **(6)**
(i) Jack and Jill went up the hill
(ii) Twenty or thirty animals were killed in the fire today.
(iii) The crop will be destroyed if there is a flood.

b) Determine whether in following is a tautology.
(i) $(7Q \wedge 7P) \wedge Q$
(ii) $P \to (P \vee Q)$
(iii) $(P$□ $2)$□ $(P \wedge Q) \vee (7P \wedge 7Q)$ **(5)**

c) Given truth values of P, Q as T and that of R, S as F. Find the truth values of the following.
(i) $(P$□ $R) \wedge (7P \to S)$
(ii) $P \vee (Q \to (R \wedge 7P))$□ $(Q \vee 7S)$ **(5)**

## UNIT-IV

8. a) In the given diagram identify the following i) an isolated vertex (node) (ii) a pair of parallel edges (iii) a loop (iv) a cycle (v) a path from $v_1$ and $v_5$ **(5)**

$e_1$ $v_1$ $e_2$ $v_2$
$e_6$ $e_7$ $e_4$ $e_3$ $v_5$
$e_8$ ∘ $v_6$
$v_4$ $e_5$ $v_3$

b) Define the following with example

(i) Strongly connected graph

(ii) Weakly connected graph

(iii) Unilaterally connected graph **(6)**

c) Let A = { w, x, y, z} and let R be a relation on A represented by the matrix.

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Construct the digraph of R and list the indegree and outdegree of each node. **(6)**

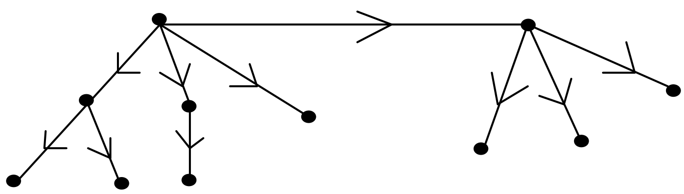
**9.** a) Define the following with one example each.

(i) Digraph

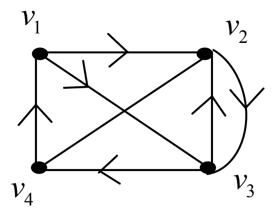
(ii) isomorphic graphs

(iii) Weighted graph **(6)**

b) Represent the following tree by a binary tree. **(6 )**



c) Define adjacency matrix of a digraph and write the adjacency matrix of the following graph. **(6)**




*******

**MAT 201.1** **Reg. No. ..................................................**

**CREDIT BASED SECOND SEMESTER B.Sc. DEGREE EXAMINATION APRIL 2015**

**MATHEMATICS**

**PAPER II: CALCULUS AND ANALYTICAL GEOMETRY**

**Duration: 3 hours** **Max Marks: 120**

**Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.**

**2. Answer FIVE full questions from Part B choosing ONE full question from each unit.**

## PART A 3x10=30

**1.** a) Find $D_x \int_0^x \sqrt{4+t^6}\,dt$

b) Find the value of $\chi$ satisfying the mean value theorem for integral for $\int_1^3 x^2 dx$

v) Find the average value of the function $f(x)=\sec^2 x$ on $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

w) Compute the length of the line segment of the line $y = 3x$ from the point (1, 3) to (2, 6).

x) Find the volume of the solid generated by revolving about $x$-axis, the region bounded by the curve $y = x^2$, the $x$ axis and the line $x = 2$.

y) Find the area of the region bounded by the graph $r = 3\sin\theta$.

z) Show that the points (5, 3, – 2), (3, 2, 1) and (– 1, 0, 7) are collinear.

aa) Find the distance between the parallel planes $2x-2y+z+3=0$ and $4x-4y+2z+5=0$

bb) Find the angle between the planes $x-2y-z=0$ and $2x-y+z=2$.

cc) Prove that the line $\frac{x-1}{2}=\frac{y-3}{3}=\frac{z-4}{-1}$ is parallel to the plane $x-2y-4z+7=0$

dd) Prove that the plane section of a sphere is a circle.

ee) Find the equation of the sphere with centre at (2, 3, 0) and which passes through the point (1, 0, 2).

ff) Identify the conic $r=\frac{3}{1-\cos\theta}$

gg) Find an equation of the graph $xy = 8$ with respect to the $\bar{x}$ and $\bar{y}$ axes after a rotation of axes through an angle of radian measure $\frac{\pi}{4}$.

hh) Find the angle through which the system has to be rotated to remove the $xy$ term from the equation $31x^2+10\sqrt{3}xy+21y^2=144$

## PART - B

### UNIT-I

2. a) Find the area of the trapezoid that is the region bounded by the lines $x = 1$ and $x = 3$, the $x$ axis and the line $2x + y = 8$. Take inscribed rectangles. **(9)**

b) State and prove the mean value theorem for integrals. **(9)**

3. a) Let the function $f$ be continuous on the closed internal [a, b] and let x be a number in [a, b]. If F is the function defined by $F(x)=\int_a^x f(t)dt$, then prove that $F^1(x)=f(x)$ **(9)**

b) Find the exact value of the integral $\int_1^3 x^2 dx$ as a limit of Riemann sum with a regular partition of the interval [1, 3] and suitable choice of $\xi_i$ **(9)**

**UNIT-II**

4. a) Find the length of the arc of the curve $y=\frac{1}{3}\left(x^2+2\right)^{3/2}$ from the point where x = 0 to the point where $x$ = 3. **(6)**

b) Find the area of the region bounded by the graph of $r=2+2\cos\theta$. **(6)**

c) Find the volume of the solid generated by revolving about the $x$ axis the region bounded by the parabola $y=x^2+1$ and the line $y=x+3$ **(6)**

5. a) Derive the length of arc of the curve $y$ = $f(x)$ in the standard form $L=\int_a^b \sqrt{1+\left(f^1(x)\right)^2}\,dx$ where $f$ and $f'$ are continuous on [a, b]. **(9)**

f) The base of a solid is the region enclosed by an ellipse having the equation $3x^2+y^2=6$. Find the volume of the solid if all plane sections perpendicular to the $x$ axis are squares. **(9)**

**UNIT-III**

6. a) Find the angle between two diagonals of a cube. **(6)**

b) Find the equation of the plane passing through the point (– 10, 5, 4) and perpendicular to the line joining the points (4, – 1, 2) and (– 3, 2, 3) **(6)**

c) Find the equation of the plane passing through the points (3, 1, 2) and (3, 4, 4) and perpendicular to the plane $5x+y+4z=0$. **(6)**

7. a) If $\theta$ is the angle between two lines with direction cosines $l_1, m_1, n_1$ and $l_2, m_2, n_2$ respectively, then prove that $\cos\theta=l_1l_2+m_1m_2+n_1n_2$ **(6)**

b) Find the bisector of the acute angle between the planes $x+2y+2z-3=0$ and $3x+4y+12z+1=0$ **(6)**

g) Find the equation of the plane which passes through the point (– 1, 3, 2) and perpendicular to the planes $x+2y+2z=5$ and $3x+3y+2z=8$. **(6)**

**UNIT-IV**

8. a) Find in the symmetrical from the equations of line of intersection of the planes $3x-2y+z-1=0=5x+4y-6z-2$, **(6)**

b) Prove that the lines $\frac{x+1}{-3}=\frac{y+10}{8}=\frac{z-1}{2}$ and $\frac{x+3}{-4}=\frac{y+1}{7}=\frac{z-4}{1}$ are coplanar. Find the point of intersection. **(6)**

c) Show that the plane $2x-y-2z=16$ touches the sphere $x^2+y^2+z^2-4x+2y+2z-3=0$, also find the point of contact. **(6)**

9. a) Find the shortest distance between the lines $\frac{x-3}{-3}=\frac{y-8}{1}=\frac{z-3}{-1}$ and $\frac{x+3}{3}=\frac{y+7}{-2}=\frac{z-6}{-4}$ **(6)**

b) Find the perpendicular distance from (3, 9, – 1) to the line $\frac{x+8}{-8}=\frac{y-31}{1}=\frac{z-13}{5}$. **(6)**

c) Show that the intersection of the sphere $x^2+y^2+z^2-2x-4y-6z-2=0$ and the plane $x+2y+2z-20=0$ is a circle. Find its centre and radius. **(6)**

**UNIT-V**

10. a) If (x, y) represents a point P with respect to the given set of axes and $(\bar{x}, \bar{y})$ is a representation of P after the axes have been rotated through an angle $\alpha$, then prove that $x=\bar{x}\cos\alpha-\bar{y}\sin\alpha$ and $y=\bar{x}\sin\alpha+\bar{y}\cos\alpha$ **(9)**

b) Simplify the equation $17x^2-12xy+8y^2-80=0$ by a rotation of axes. Also draw the sketch of the graph of the equation. **(9)**

11. a) Remove the $xy$ term from the equation $24xy-7y^2+36=0$ by rotating the axes. **(6)**

b) An equation of a conic is $r=\frac{6}{4+5\sin\theta}$. Find the eccentricity, identify the conic, write an equation of the directrix corresponding to the focus at the pole, find the vertices and draw a sketch of the curve. **(6)**

c) A parabola has its focus at the pole and its vertex at $(4, \pi)$. Find an equation of the parabola and an equation of the directrix. Draw a sketch of the parabola and the directrix. **(6)**

*******

MAT 201.2 **Reg. No. ................................................**

**CREDIT BASED SECOND SEMESTER B.Sc. DEGREE EXAMINATION APRIL 2015**

**MATHEMATICS**

**PAPER II: NUMBER THEORY AND DIFFERENTIAL EQUATIONS**

**Duration: 3 hours** **Max Marks: 120**

**Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.**
**2. Answer FIVE full questions from Part B choosing ONE full question from each unit.**

**PART A** **3x10=30**

**1.** a) If p is a prime and p | ab then prove that p | a or p | b

b) Find the remainder when $1!+2!+...+99!+100!$ is divided by 12.

ii) Solve the linear congruence $6x \equiv 15 \pmod{21}$

jj) If p is a prime then prove that $a^p \equiv a$ (mod p) for any integer $a$

kk) Find $\phi(36000)$

ll) Find the last two digits in the decimal representation of $3^{256}$ using Euler's theorem, given that $\phi(100) = 40$

mm) Express $[3;2,1,2,5,1]$ as a rational number

nn) If $\{u_n\}$ is a Fibonacci sequence then what is the $\gcd(u_{16}, u_{12})$

oo) If x, y, z is a primitive Pythagorean triple, then prove that one of the integers x and y is even while the other is odd. .

pp) Check the exactness of the equation $(2xy - \tan y)\,dx + (x^2 - x sec^2 y)dy = 0$

qq) Solve $y(2xy+1)dx - xdy = 0$

rr) Find the integrating factor of the differential equation $y' = \cos ecx + y\cot x$

ss) Solve: $x^2p^2 - y^2 = 0$

tt) Solve: $y = px + p^3$

uu) Obtain the p-discriminant of the equation $p^2 - xp - y = 0$

**PART - B**

**UNIT-I**

2. a) Prove that every positive integer n > 1 can be expressed as a product of primes, and also prove that this representation is unique apart from the order in which factors occur. **(6)**

b) Let $N = a_m \cdot 10^m + a_{m-1} \cdot 10^{m-1} + a_{m-2} \cdot 10^{m-2} + ... + a_1 \cdot 10 + a_0$ be the decimal representation of the positive integer N, $0 \le a_m < 10$ and let $T = a_0 - a_1 + a_2 - a_3 + ... + (-1)^m a_m$. Prove that $11 | N$ if and only if $11 | T$. **(6)**

c) Prove that $a \equiv b$ (mod n) if and only if $a$ and b leave the same nonnegative remainder when divided by n. **(6)**

3. a) Prove that the linear congruence $ax \equiv b$ (mod n) has a solution if and only if $d \mid b$ where $d = \gcd(a, n)$. Also if $d \mid b$ then prove that it has d mutually incongruent solutions modulo n. **(6)**

b) Solve the simultaneous linear congruences

$x \equiv 2 \pmod{3}$

$x \equiv 3 \pmod{5}$

$x \equiv 2 \pmod{7}$ **(6)**

c) If $ca \equiv cb \pmod{n}$ then prove that $a \equiv b \left( \bmod \frac{n}{d} \right)$ where $d = \gcd(c, n)$ **(6)**

## UNIT-II

4. a) If $p$ is a prime and $p$ does not divide $a$ then prove that $a^{p-1} \equiv 1 \pmod{p}$ **(9)**

b) Prove that the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$ where $p$ is an odd prime, has a solution if and only if $p \equiv 1 \pmod{4}$ **(9)**

5. a) If $p$ is a prime then prove that $(p-1)! \equiv -1 \pmod{p}$ **(9)**

h) If n is a positive integer and $\gcd(a, n) = 1$, then prove that $a^{\phi(n)} \equiv 1 \pmod{n}$ where $\phi(n)$ is Euler's phi function **(9)**

## UNIT-III

6. a) Prove that the kth convergent of a simple continued fraction $[a_0; a_1, a_2, ....a_n]$ has the value $C_k = \frac{p_k}{q_k}, 0 \le k \le n$ where $p_k = a_k p_{k-1} + p_{k-2}$ and $q_k = a_k q_{k-1} + q_{k-2}$ for $k \ge 2$ **(6)**

b) Prove that the radius of the inscribed circle of a Pythagorean triangle is always an integer. **(6)**

c) Express $\frac{187}{57}$ as a finite simple continued fraction **(6)**

7. a) For $m \ge 1, \quad n \ge 1$ prove that $U_{mn}$ is divisible by $U_m$ **(6)**

b) Prove that the area of a Pythagorean triangle can never be equal to a perfect (integral) square **(6)**

i) Prove that any rational number can be written as a simple continued fraction. **(6)**

**UNIT-IV**

8. a) Solve: $\left(y^2 - 2xy + 6x\right)dx - \left(x^2 - 2xy + 2\right)dy = 0$ **(6)**

b) Determine the integrating factor and solve $\left(4xy + 3y^2 - x\right)dx + x(x + 2y)dy = 0$ **(6)**

c) Radium decomposes at a rate proportional to the quantity of radium present. Suppose that it is found that in 25 years approximately 1.1 percent of a certain quantity of radium has decomposed. Determine approximately how long it will take for one-half of the original amount of radium to decompose. **(6)**

9. a) Solve $y' = y - xy^3e^{-2x}$ **(6)**

b) Solve $(x + 4y - 9)dx + (4x + y - 2)dy = 0$ **(6)**

c) Find the orthogonal trajectories of the family of curves given by $r = a(1 + \sin\theta)$ **(6)**

**UNIT-V**

10. a) Solve $xp^2 - (2x + 3y)p + 6y = 0$ **(6)**

b) Find the general and singular solution of $x^2p^2 + 3xp + 9y = 0$ **(6)**

c) $yy'' + \left(y'\right)^2 + 1 = 0$ **(6)**

11. a) Solve: $xp^2 + \left(1 - x^2y\right)p - xy = 0$ **(6)**

b) Find the general solution and singular solution of $p^2 + 4x^5p - 12x^4y = 0$ **(6)**

c) Solve: $xy'' - (y')^3 - y' = 0$ **(6)**

******

MAT 202 **Reg. No. ..............................................**

**CREDIT BASED SECOND SEMESTER B.C.A. DEGREE EXAMINATION APRIL 2015**

**MATHEMATICS**

**PAPER II: FUNDAMENTALS OF MATHEMATICS**

**Duration: 3 hours** **Max Marks: 80**

**Note: 1. Answer any SIX questions in Part A. Each question carries 2 marks.**
**2. Answer any FOUR full questions from Part B choosing ONE full question from each unit.**

**PART A** **2x6=12**

**1.** a) Represent the following using Venn diagram (i) □ $(A \cup B)$ (ii) □ $A \cap B$

b) Write the power set of A if A = {1, 2, 3}.

vv) If f and g are two functions on the set of real numbers defined by $f(x) = x - 2$ and $g(x) = x + 4$ find $f \circ g$.

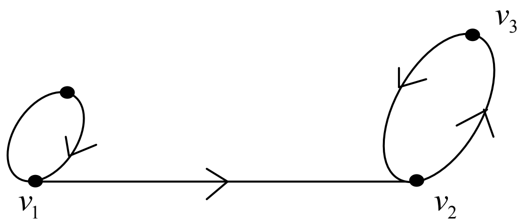
ww) Define characteristic function of a set A.

xx) Construct the truth table for the compound proposition $P \wedge (P \vee Q)$.

yy) Write in symbolic form "The crop will be distroyed if there is a flood".

zz) What is length of a path? Explain with an example.

aaa) Write the in-degree and out-degree of each of the nodes in the following graph.




**PART - B**

**UNIT-I**

2. a) If A = {1, 2, 4, 6, 8}, B = {2, 4, 5, 9} and
C = {x | x is a positive integer and $x^2 \leq 16$},
find (i) $A \cap (B \cup C)$ (ii) $(A \cup B) \cap (A \cup C)$ **(5)**

b) Define equivalence relation. Find an equivalence relation on the set
A = (1, 2, 3, 4}. **(6)**

c) If $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

are two relation matrices find $M_{R\circ\tilde{S}}$, $M_{R\tilde{\circ}S}$ and $M_{\tilde{S}\circ\tilde{R}}$ **(6)**

3. a) If A = {x | x is an integer and $0 \le x \le 3$} and B = {4, 5}, find the following.
(i) $A \times B$ (ii) number of elements in $P(A \times B)$
(iii) number of relations from A to B. **(6)**

b) If X = {1, 2, 3, 4} and
R = {<1, 1>, <1, 4>, <4, 1>, <4, 4>, <2, 2>, <2, 3>, <3, 2>, <3, 3>} write the matrix of R and sketch the graph. **(6)**

c) Let X = {2, 3, 6, 12, 24, 36} and the relation '$\le$' be such that $x \le y$ if x divides y. Draw the Hasse diagram of $(X, \le)$ **(5)**

## UNIT-II

4. a) Let f and g be two functions on the set of real numbers defined by $f(x) = x^2 - 2$ and $g(x) = x + 4$. Show that $g(x)$ is a bijective function. Also find $g \circ f \circ f$. **(6)**

b) Define inverse of a function. If $f : R \to R$ is defined by $f(x) = x^3 - 2$ find $f^{-1}$ **(5)**

c) Let * be a binary operation on a set X which is associative and has the identity element 'e'. If an element $a \in X$ is invertible, then prove that both left inverse and right inverse of $a$ are equal. **(6)**

5. a) Let A = {1, 2, 3), f and g are functions on A given by f = {<1, 2>, <2, 2>, <3, 4>, <4, 1> and g = {<1, 3), <2, 1>, <3, 3>, <4, 3>}, then find the following
(i) $f \circ g$ (ii) $g \circ f$ (iii) $g \circ g$ **(5)**

j) Define characteristic function of a set and using the properties of characteristic functions prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **(6)**

k) Let $f : z \times z \to z$ be defined by $f(x, y) = x * y = x + y - xy$ show that the operation * is commutative and associative. Also find the identity element under * **(6)**

## UNIT-III

6. a) Show that $\left(\neg P \wedge (\neg Q \wedge R)\right) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ **(6)**

b) Define tautology and contradiction. Find whether $(P \vee Q) \wedge (P \wedge Q)$ is a tautology or not. **(5)**

d) Give the truth values of P and Q as T and that of R and S are F. find the truth value of the following.
(iii) $\left(\neg(P \wedge Q) \vee \neg R\right) \vee \left(Q \square \neg P\right) \to R \vee (\neg S)$
(iv) $(P \vee (Q \to (R \wedge \neg P) \square (Q \vee \neg S)$ **(6)**

7. a) Write the truth table for the following **(5)**
(i) Disjunction (ii) Conditional (iii) Biconditional

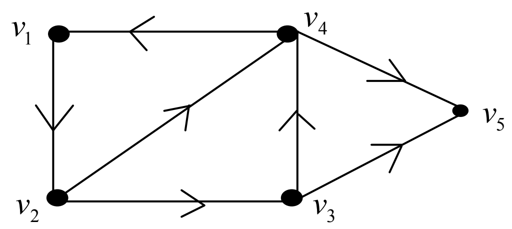
b) Construct truth table for the following implications.
(i) $Q \wedge (P \to Q) \to P$
(ii) $\neg(P \vee (Q \wedge R))$ □ $(P \vee Q) \wedge (P \vee R)$ **(6)**

c) Prove that $\left((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))\right) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$
is a tautology **(6)**

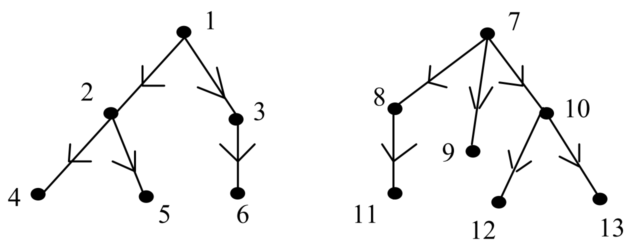
**UNIT-IV**

8. a) Explain the following with example
(i) isolated vertex (ii) parallel edges (iii) Indegree and outdegree **(6)**

b) Define reachable set of a set S of nodes in a digraph. Also find the reachable set of the sets $\{v_1, v_4\}$, $\{v_4, v_5\}$ and $\{v_3\}$ in the following graph **(6)**




c) Convert the following forest into a binary tree **(5)**

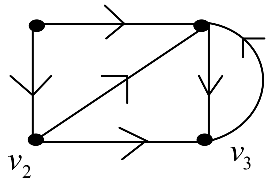



9. a) Define the following.
(i) Path
(ii) Path length
(iii) Simple path
(iv) Elementary path
(v) Cycle **(6)**

b) If $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ is the adjacency matrix of a digraph G, then represent the digraph $\overline{G}$, the converse of G, graphically. **(6)**

c) Define node base in a digraph and find the node base of the following graph **(5)**

$v_4$
$v_1$


$v_2$
$v_3$

*******

11. a) Find the p-discriminant and those solutions of the differential equation that are contained in the p-discriminant of $xp^2 - 2yp + 4x = 0$ **(6)**

b) Solve $(x^2-1)p^2 - 2xyp + y^2 - 1 = 0$ by reducing it to Clairaut's form. **(6)**

c) Solve: $yy'' + (y')^2 + 1 = 0$ **(6)**

******

MAT 201.2 Reg. No. ..............................................

**CREDIT BASED SECOND SEMESTER B.Sc. DEGREE EXAMINATION APRIL 2016**

**MATHEMATICS**

**PAPER II: NUMBER THEORY AND DIFFERENTIAL EQUATIONS**

**Duration: 3 hours** **Max Marks: 120**

**Note: 1. Answer any TEN questions in Part A. Each question carries 3 marks.**
**2. Answer FIVE full questions from Part B choosing ONE full question from each unit.**

## PART A 3x10=30

**1.** a) For arbitrary integers $a$ and $b$, prove that if $a \equiv b(\mod n)$ then $a$ and $b$ leave the same remainder on division by $n$.

b) Find the remainder obtained when $2^{20}$ is divided by 41.

c) Solve $9x \equiv 21 \pmod{30}$

d) If p and q are distinct primes with $a^p \equiv a(\mod q)$ and $a^q \equiv a(\mod p)$ then prove that $a^{pq} \equiv a(\mod pq)$

e) Find $\phi(5040)$

f) Using Euler's theorem find the last two digits in the decimal representation of $3^{256}$ given that $\phi(100) = 40$

g) Find the rational number represented by [0; 2, 1, 2, 5, 1]

h) For the Fibonacci sequence $\{u_n\}$, prove that $\gcd(u_n, u_{n+1}) = 1$ for every $n \geq 1$.

i) If x, y, z is a primitive Pythagorean triple, then prove that one of the integers x and y is even while the other is odd.

j) Determine whether the function $f(x,y) = x\sin\frac{y}{x} - y\sin\frac{x}{y}$ is homogeneous or not. If it is homogeneous find its degree.

k) Solve $y(2xy+1)dx - xdy = 0$

l) Check the exactness of the equation $\left(y^2 - 2xy + 6x\right)dx - \left(x^2 - 2xy + 2\right)dy = 0$

m) Obtain the p-discriminant of equation $p^2 - xp - y = 0$

n) Solve $xyp^2 + (x+y)p + 1 = 0$

o) Solve $y = px + kp^2$

## PART - B

### UNIT-I

2. a) Prove that every positive integer n > 1 can be expressed as a product of primes, and also prove that this representation is unique apart from the order in which factors occur. (6)

b) Let $N = a_m 10^m + a_{m-1}10^{m-1} + ...... + a_0$ be the decimal expansion of the positive integer N, $0 \le a_k < 10$ and let $S = a_0 + a_1 + ........ + a_m$. Then prove that $9 | N$ if and only if $9 | S$. (6)

c) Find the solution of the system of congruences

$x \equiv 2 \pmod 3$
$x \equiv 3 \pmod 5$
$x \equiv 2 \pmod 7$ (6)

3. a) Let $n_1, n_2, ......n_r$ be positive integers such that $\gcd(n_i, n_j) = 1$ for $i \ne j$. Then prove that the system of linear congruences

$x \equiv a_1 \pmod{n_1}$
$x \equiv a_2 \pmod{n_2}$
.
.
.
$x \equiv a_r \pmod{n_r}$

has a simultaneous solution which is unique module integer $n_1 n_2 ......n_r$ (6)

b) If $ca \equiv cb \pmod n$ then prove that $a \equiv b \left( \bmod \frac{n}{d} \right)$ where $d = \gcd(c, n)$ (6)

c) If p is a prime and $p | ab$ then prove that $p | a$ or $p | b$ (6)

### UNIT-II

4. a) If $p$ is a prime and $p$ does not divide $a$ then prove that $a^{p-1} \equiv 1 \pmod p$ (9)

b) If n is a positive integer and $\gcd(a, n) = 1$, then prove that $a^{\phi(n)} \equiv 1 \pmod n$ where $\phi(n)$ is Euler's phi function (9)

5. a) If $p$ is a prime then prove that $(p-1)! \equiv -1 \pmod p$ (9)

b) Prove that the quadratic congruence $x^2 + 1 \equiv 0 (mod\ p)$ where p is an odd prime has a solution if and only if $p \equiv 1 \pmod 4$ (9)

### UNIT-III

6. a) For $m \ge 1$, $n \ge 1$ prove that $u_{mn}$ is divisible by $u_m$ (6)

b) If $C_k = \frac{p_k}{q_k}$ is the $k^{th}$ convergent of the simple continued fraction $[a_0; a_1, a_2, ..., a_n]$ then prove that $p_k q_{k-1} - q_k p_{k-1} = (-1)^{k-1}, 1 \le k \le n$. where $p_k = a_k p_{k-1} + p_{k-2}$ and $q_k = a_k q_{k-1} + q_{k-2}$ for $k \ge 2$ (6)

c) Express $\frac{170}{53}$ as a finite simple continued fraction. (6)

7. a) Prove that the area of a Pythagorean triangle can never be equal to a perfect (integral) square (6)

b) If $m = qn + r$ then prove that $\gcd(u_m, u_n) = \gcd(u_r, u_n)$ (6)

c) Prove that any rational number can be written as a simple continued fraction. (6)

### UNIT-IV

8. a) Solve $3x(xy-2)dx + (x^3 + 2y)dy = 0$ (6)

b) Determine the integrating factor and solve $y(x+y+1)dx + x(x+3y+2)dy = 0$ (6)

c) A thermometer reading $18^\circ F$ is brought into a room where the temperature is $70^\circ F$. One minute later the thermometer reading is $31^\circ F$. Determine the temperature reading as a function of time. (6)

9. a) Solve $(1 + 3x\sin y)dx - x^2 \cos y dy = 0$ (6)

b) Solve $(x + 2y - 4)dx - (2x + y - 5)dy = 0$ (6)

c) Find the orthogonal trajectories of the family of curves given by $r = a(1 + \sin\theta)$ (6)

### UNIT-V

10. a) Solve $x^2 p^2 - 5xyp + 6y^2 = 0$ (6)

b) Find the general and singular solution of $xp^2 - 3yp + 9x^2 = 0$ for $x > 0$ (6)

c) Solve $xy'' - (y')^3 - y' = 0$ (6)